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Skills, agglomeration and segmentation

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Abstract

This paper studies the role of skill heterogeneity in "new economic geography" models of location. In our setting, products are both horizontally and vertically differentiated, and producing higher quality goods requires workers with higher skills. Selling to customers based in a different location entails iceberg-type transport costs and additional "communication costs" consisting of a fixed quality loss. We show that the presence of pecuniary externalities creates a mechanism which *always* promotes spatial sorting of workers according to their skill levels. In particular, in *all* stable equilibria, workers with higher skill choose to stay in the location where aggregate skill and income is higher, while the less skilled stay in the other.

JEL classification: R12; R13; F12; F16

Keywords: Skill heterogeneity; Agglomeration; Core–periphery model; Regional inequality; Interpersonal inequality; Transport costs

1. Introduction

Economic activities are not evenly distributed in space. Some places are crowded with firms and workers, while others, poor of people and human capital, lag behind. The economic landscape is full of humps and bumps, that result from an unequal distribution of labor across space, both in terms of its quantity and quality. We generally observe that the most educated and talented workers cluster together in wealthy and economically important areas, while low skilled workers are more often located in low

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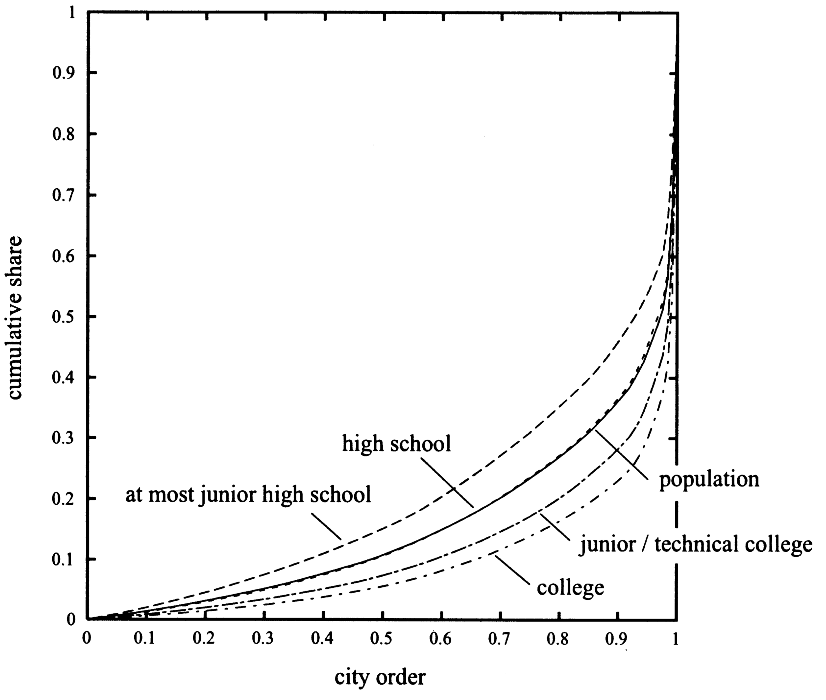

Fig. 1. Lorenz curves of inter-city inequality in education level.

income areas. Which factors account for the observed agglomeration of workers, and for their spatial segmentation across skill and income levels? This paper studies the role of skill heterogeneity in "new economic geography" models of location. In particular, we show that, in the presence of pecuniary externalities based on product variety and quality, there is a mechanism which *always* promotes spatial sorting of workers according to their skill levels, providing results consistent with available evidence.

Workers tend to cluster together, the more so, the higher their level of human capital. Fig. 1 helps to illustrate the point. There, we report the Lorenz curves of the inter-city distribution of workers in Japan, year 1990.[1] Lorenz curves are depicted separately for the total population and for the workers belonging to particular education categories.[2] The figure clearly indicates that workers with higher education levels

[1] *Data source*: Japan Statistics Bureau (1990). A city here is a Standard Metropolitan Employment Area (SMEA) which is an aggregation of counties based on the commuting pattern. For the precise definition of SMEA, see Yamada and Tokuoka (1983).

[2] The horizontal axis ranks cities in terms of their shares of workers with a given education level (the rank is normalized by the total number of cities, 124). The vertical axis reports the cumulative share of workers of a given education level in the cities up to a given position in the rank.

are more geographically concentrated. [3] The data also show that workers with higher education are more easily found in big locations: the rank correlation between city size and the share of college graduates in a city is 0.73, while that between city size and the share of workers with at most junior high school degree is −0.67. Tokyo alone accounts for more than 40% of college graduates among all metropolitan areas. These patterns are not typical of Japan: similar evidence is obtained for the case of the US. [4]

The lumpy inter-regional distribution of human capital results from the location decisions of workers, which may differ according to their skill levels (based on their education, experience and innate ability). The existing distribution of skill affects in turn the outcome of the migration choices of workers. [5] The existing theoretical explanations for the spatial sorting of workers according to their skill levels are based on the existence of agglomeration forces generated by human capital externalities (Black, 1999; Black and Henderson, 1999). [6] In these models, from the possible locations (interpreted as cities), each worker choose one where other workers are located, since this way he/she can benefit from knowledge spillovers, and obtain higher productivity and a higher wage rate. However, the geographic concentration of population is assumed to cause also urban costs, related to land scarcity and congestion. Workers are heterogenous in terms of their human capital endowments and may judge differently where to locate. A crucial assumption of these models is that the human capital externalities have a greater impact on workers with higher skills. The predicted result is therefore that the workers with high skill will concentrate in the same location, bidding out less skilled ones. [7]

The aim of this paper is to show that the uneven spatial distribution of skill may be explained also in a general spatial setting, without relying on human capital externalities and congestion costs, which are likely to be relevant when locations are thought in terms of cities or urban areas. In our model, the centripetal and centrifugal forces in space originate from pecuniary externalities arising from the interaction of increasing returns

[3] The value of the Gini index for the distribution of the total population is 0.69, while the corresponding value for the cases of workers with education up to junior high school, high school, junior/technical college, and college, is 0.60, 0.69, 0.77, and 0.82, respectively.

[4] In 1990, the Gini index value for the inter-city distribution of workers is 0.61, while it is 0.66 for the case of college graduates (aged above 25). The rank correlation between city size and college graduates share in the city workforce is 0.40. These figures are calculated from Black (1999)'s data set which is based on the Population Census of the US.

[5] Borjas et al. (1992) and Rauch (1992) give empirical support to the view that these patterns may be due to the workers' tendency to self-select according to their skill levels across locations. The difference between one's own skill level and the average skill level of one's initial home town appears to be indeed an important explanatory factor for individual migration decisions.

[6] See also Fernandez (2001) for a review of models that explain intra-city spatial sorting according to skill levels on the ground of heterogeneity in preferences for local public goods.

[7] In the presence of urban costs as in Black (1999) and Black and Henderson (1998), Abdel-Rahman (1998) shows the possibility of inter-city segementation between skilled and unskilled workers without assuming any positive externality. However, no explicit condition for the segmentation is obtained.

to scale with transport costs.[8] In the presence of pecuniary externalities, economic activities tend to concentrate in a limited number of locations due to the operations of cumulative processes related to the existence of demand ("backward") and cost ("forward") linkages. The story goes as follows. Producers will choose locations that have good access to large markets and to supplies of inputs they may require. In turn, a location in which for some reason there is already a vast presence of producers tends to offer also a large market for their goods and a thick supply of intermediate inputs. Thus, on one hand, this circular causation generates economies of agglomeration, while on the other hand, there are costs of agglomeration, in particular, due to tougher competition among co-located producers. We show that when the skill levels of agents are not equal, spatial sorting according to skill levels arise as a general phenomenon. Agents with low skill may not be able to endure the tough competition at large agglomerations, and may rather avoid these locations. Conversely, since the highest skilled care less about local competition, they will seek for the locations where they can fully exploit agglomeration economies.[9]

Our model builds on the "core–periphery" model by Krugman (1991) which offers an analytically convenient representation of how the interaction among scale economies, transport costs and an immobile source of demand generates agglomeration/dispersion of mobile agents through pecuniary externality. We modify this core–periphery model in two ways. First, the existing versions of the core–periphery model do not account for the location patterns of economic agents with different skill levels, leaving unexplained the observed tendency towards spatial segmentation according to skill levels. In contrast, the economy we have in mind is made up of many heterogenous worker-sellers who offer goods that are differentiated both horizontally (variety) and vertically (quality), where quality requires skills. The earnings of mobile worker-sellers consist of rents associated with their skills.

The second modification is on the structure of transport costs. In the core–periphery model, transport costs of "iceberg" type are assumed, meaning that selling to distant markets entails a "physical" transport cost, i.e., a given fraction of the shipped good

[8] The interaction between scale economies and transport costs as a source of agglomeration has a long-standing tradition in spatial economics. Surveys can be found in Fujita (1990) and Fujita et al. (1999). As for empirical evidence on the origin of agglomeration forces in reality, it seems quite difficult to disentangle empirically between the different sources of externalities that explain the observed agglomeration patterns. Davis and Weinstein (1999) find evidence of a significant "home market effect," namely, a more than proportional relation between local production and local demand implied by the existence of scale economies and factor mobility. Rauch (1992) presents results in favor of the existence of knowledge spillovers, showing that the US workers are more productive in cities where average levels of educational attainment are higher. Results similar to Rauch (1992) are found in Glaeser et al. (1995), Ciccone and Hall (1996), and Dobkins and Ioannides (2000), while Ciccone (2002), Ciccone and Peri (2000) and Ciccone et al. (1999) reject the hypothesis of significant human capital externalities. Finally, Dumais et al. (2002) find empirical confirmation of both pecuniary externalities and knowledge spillovers, but claim that labor market pooling have the major role in explaining agglomeration patterns.

[9] Abdel-Rahman and Wang (1995) also introduce the pecuniary externality (matching externality in skilled labor market) to explain inter-city segmentation between skilled and unskilled. However, the mechanism of the segmentation is rather ad hoc, since the positive externality is assumed only when skilled agglomerate in the same city.

melts away. Under this assumption, the transport cost is multiplicative to the price of shipped good, which in turn implies that selling to a distant location corresponds to a constant share of sales (and profits) lost in transit. Since transport costs and the value of the shipped goods are proportional, the price elasticity of demand is the same irrespective of distance between sellers and customers. This greatly improves analytical tractability of the model, and is the practical motivation of the iceberg assumption. This also means that the demand for products is affected in the same proportion by distance irrespective of their quality. In reality, however, it is often the case that the impact of distance on demand depends on the quality of products in such a way that the sales of the more skilled sellers will be hit relatively less by distance from the market. This is consistent with the observation that only the most reputed firms and the most talented professionals start selling their goods and services in faraway markets.

In our model, we assume that the effect of transport costs increases less proportionally with the value of the shipped goods. This permits to obtain a different impact of distance on sellers endowed with different skills and supplying goods of different quality levels. To this end, in addition to iceberg transport costs, we introduce a second cost associated with distance, consisting of a quality depreciation term, common to all goods consumed in a remote location. [10] We name these "communication costs." [11] In this setting, the geographic provenience of goods has a direct consequence on consumer welfare, and then on demand. Since the quality depreciation term is the same for all shipped goods, the loss is relatively small for higher-quality goods, so that the demand for them is less affected. This, in turn, explains a different location behavior for workers endowed with different skill levels. Those sellers that supply goods of higher quality are relatively footloose, since their sales can penetrate distant markets easily. [12]

In this context, the key determinants of agents' location decisions are the degree of local competition (the seller viewpoint) and that of product availability, summarized by the "true" cost of living index (the consumer viewpoint) at each location. The spatial distribution of population and skill, in turn, affects the extent of competition and the cost of living in different locations. Since the sales and incomes of worker-sellers with higher skills are less affected by distance, their location decisions will be based mainly by considerations related to variety size and quality of the goods available in alternative locations. Conversely, the less skilled are more affected by distance, and their location

[10] An alternative approach to obtain a different impact of distance across sellers would be to assume additive monetary transport costs that are, for instance, linear in the quantity of the good shipped. However, adopting such an approach in our analysis while keeping tractability of the model would sacrifice other parts of the model structure (see, e.g., Chapter 3 in Ottaviano, 1998).

[11] One can think of many instances in which imperfect communication between distant buyers and sellers is a source of costs and reduced trade. Buyers may need locally provided information to fully appreciate some goods. Alternatively, the use of some products may require some interaction between buyers and sellers, and this is made difficult by distance. In general, we observe empirically a strong "home bias" in consumption: transport and trade costs are not enough to explain the preference of consumers for home-made goods (see, e.g., Helliwell, 1997).

[12] In principle, we can simply replace iceberg transport costs by communication costs to realize the non-proportionality between the costs for shipping and the value of shipped goods, and obtain the same basic results. However, as will become clear in Section 2.2, it is analytically more demanding than to assume the iceberg costs as the major part of shipping costs.

decisions are mainly dictated by the degree of local competition. Thus, starting with even a slightly unequal inter-regional distribution of population and skill, this location mechanism makes more skilled and less skilled seek for opposite locations, enhancing this way the initial imposed asymmetry between locations. This occurs because of a quite simple reason: the region with a greater (resp., lower) mass of skill will be both the location where competition is stronger (resp., weaker) and the one where goods with a higher (resp., lower) level of quality are available.

For the above reasons, we show that in *all* stable equilibria, mobile worker-sellers are sorted between the regions according to skill levels: the more skilled choose to stay in the location where aggregate skill and income are higher, while the less skilled stay in the other. Agents' heterogeneity in the presence of pecuniary externalities works as a source of inevitable regional inequality. Workers' segmentation across skill levels can thus be thought of as a pervasive tendency that does not necessarily require the presence of human capital externalities. Market interactions among unequal agents can be sufficient to generate spatial sorting according to skill levels.[13]

The remainder of the paper is organized as follows. The next section presents the structure of the model. Section 3 introduces the notion of temporary equilibrium and adjustment process. Section 4 defines the equilibrium. Section 5 classifies possible equilibrium configurations, and is devoted to the analysis of their existence and stability. Section 6 discusses some implications of our model. The concluding comments end the paper.

2. The model

2.1. The economy

The economy consists of two regions which are symmetric except for the mass of skilled workers that are located in each. Variables in the model bear a subscript $r = a, b$ to indicate the region. Firms produce differentiated goods under imperfect competition and free-entry. Consumers (skilled and unskilled workers) like variety, according to the Dixit–Stiglitz (1977) formulation. There are two types of primary production factors: skilled and unskilled labor. Each worker embodies a unit of corresponding labor. Production requires two inputs: skill ("talent") provided by the former and an intermediate input, produced out of the latter. Skilled workers are perfectly mobile between the regions, while the unskilled are assumed to be immobile. The markets for the production factors and the intermediate good are competitive.

Unskilled workers are homogeneous. A unit mass of them is located in each of the two regions. Skilled workers are heterogeneous, in that they are distinguished by different skill levels. There is a unit mass of skilled workers in the whole economy. Each skilled worker is characterized by her skill level s; a more talented worker is

[13] The possibility of international inequality has been already explained on the pure ground of pecuniary externalities. Matsuyama (1996) proposes an argument based on increasing returns arising from the trade in inputs and the international division of labor.

associated with a larger value of s. The lowest and the highest skill levels among all skilled workers are denoted, respectively, by $\underline{s}$ and $\bar{s}$, where $0<\underline{s}<\bar{s}<\infty$. Skilled workers are distributed over the interval, $S\equiv[\underline{s},\bar{s}]$, according to the exogenous density function $f(s)$. The aggregate (and average) skill in the economy is denoted by $\hat{s}$, which equals $\int_{s\in S} sf(s)\,\mathrm{d}s$. No additional assumptions are made on the distribution of skills. To characterize the regional distribution of workers of each skill level, let $f_r(s)$, $r=a,b$, represent the density of workers with skill level s in region r and let n_r and s_r denote, respectively, *the share of skilled population*, and *the share of aggregate skill* in region r. Note that the distribution $f_r(s)$, $r=a,b$, is endogenous, i.e., it is determined at equilibrium. Then we can write:

$$n_r=\int_{s\in S} f_r(s)\,\mathrm{d}s, \quad s_r=\frac{1}{\hat{s}}\int_{s\in S} sf_r(s)\,\mathrm{d}s, \tag{1}$$

where

$$s_a+s_b=1, \quad n_a+n_b=1, \tag{2}$$

$$f_a(s), f_b(s)\geqslant 0 \quad \text{and} \quad f_a(s)+f_b(s)=f(s) \quad \text{for each } s. \tag{3}$$

The intermediate input is produced through a linear technology using unskilled labor as sole input. Final production requires the services of skilled labor and a given amount of the intermediate input proportional to output. While the intermediate inputs provide standardized production services, skilled workers' talent adds "value," or "quality," to each unit of the final good. Goods of higher quality are more appreciated by consumers. Consequently, we consider products that are differentiated along the horizontal dimension (variety) and the vertical one (quality). [14]

The intermediate input is costlessly mobile across regions. Shipping final goods instead requires paying iceberg transport costs: a fraction of the good is lost in transit. We further assume that consuming final goods outside the region in which they are produced entails an additional communication cost. Namely, the perceived quality of products is lower if the goods are shipped to the other region.

2.2. Technology and preferences

The consumption good can be differentiated along a continuum of varieties $i\in R$. Each variety i is produced out of intermediate inputs and skill according to the same technology. The size of firms is normalized in such a way that one firm employs one skilled worker only. [15] We further assume that each skilled worker can employ her skill in the production of at most one variety of the consumption good. Let $w_r(s)$ and

[14] The same formulation is found in Manasse and Turrini (2001).

[15] An alternative interpretation is that each worker is running a firm. In the remainder of the paper we refer to our mobile agents as workers or worker-sellers.

υ_r denote, respectively, the return to the skill of a worker endowed with "talent" s and the marginal cost (consisting of expenses for intermediate inputs) of final production in region r. Then, the cost $C_r(Q, s)$ of producing Q units of any variety in region r by employing a worker with skill level s is given by

$$C_r(Q, s) = w_r(s) + \upsilon_r Q. \tag{4}$$

Firms are atomistic profit-maximizers. They produce goods which are imperfect substitutes and set their price taking as given other firms' choices (the "large group" Chamberlinian hypothesis holds). Consumer utility increases with the extent of variety in consumption. As it is standard in monopolistic competition models, love for variety plus increasing returns in production insure that no firm is willing to supply the same variant offered by a rival. Since each firm requires the skill of one worker, we necessarily have the total variety size equal to 1. In turn, the condition of free-entry ensures that *skilled workers receive all the operating profits realized by firms*. Skilled workers' income thus consists of skill rents associated with firms' operating profits.

As for the production of intermediate inputs, we simply assume that one unit of unskilled labor produces one unit of the intermediate input. It follows from the assumption of perfect competition in the market of intermediates that wages for unskilled workers in region r equal υ_r.

We now turn to the problem of consumers. Recall that in our formulation more talented workers produce a good of better quality, and better quality is appreciated by consumers. For simplicity, and without affecting our qualitative results, we assume that the skill level of the worker employed for the production of the good exactly conveys the quality of the good. So, hereafter we use the terms, skill level and quality, interchangeably.

Consumers derive utility from a combination of the quantity of each variety i, $x(i)$, and the perceived quality of the good of variety i. Because of love for variety, individuals will smooth their consumption across all available varieties, both domestic and imported. However, in the case of shipped varieties, the perceived quality level is lower. While for the case of domestic goods the perceived quality is equal to the skill level $s(i)$ associated with the good, when the good is imported the perceived quality equals $s(i) - c$, where $c > 0$ represents the communication cost. So, the geographic provenience of goods has a direct consequence on consumer welfare.[16] As will be clear in the following analysis, the presence of communication costs plays a crucial role. The reason is that *introduces a non-convexity in the technology for selling goods to distant locations*. This non-convexity shows up across sellers, with more skilled agents selling larger amounts of their goods, being proportionally less affected by distance compared with the less skilled.

A Cobb–Douglas specification is chosen to nest the quantity $x(i)$ and the quality $s(i)$ in consumers' utility function, while a standard CES specification is used to nest

[16] Notice the analogy of this specification of communication costs with that used in many models aimed at explaining location patterns on the ground of agents' need for direct contact and face-to-face interaction. See Fujita and Smith (1990) for a comprehensive survey.

different varieties.[17] Denoting the set of varieties produced within and outside region r by N_r and N_{-r}, respectively, the utility level of a consumer located in region r is given by

$$u_r = \left[\int_{i \in N_r} s(i)^{1/\sigma} x(i)^{(\sigma-1)/\sigma} \, \mathrm{d}i + \int_{i \in N_{-r}} (s(i) - c)^{1/\sigma} x(i)^{(\sigma-1)/\sigma} \, \mathrm{d}i \right]^{\sigma/(\sigma-1)}. \tag{5}$$

As usual, $\sigma > 1$ is the elasticity of substitution across different varieties. Since firms are atomistic, σ is also the price elasticity of the demand for each variety.

Unlike the models based on the transport cost of only the iceberg type, in our model, the goods are not traded if the communication costs are too high ($c > s$). Since the analysis in this case is unnecessarily involved, we will focus on the case in which all varieties are traded at equilibrium. Namely, we maintain the following assumption:[18]

Assumption 1. $0 < c < \underline{s}$.

Finally, notice that our model has qualitatively the same structure as that of the original core–periphery model if all mobile agents are homogeneous and communication costs are zero.

2.3. *Distribution of skills and feasible spatial configurations*

In our economy, the interpersonal distribution of skill and the spatial distribution of skilled workers are necessarily related. A graphical representation of the economy as in Fig. 2 helps the intuition.[19] In the graph, the horizontal distance from O (resp., O') measures the (skilled) population share in region a (resp., b). The vertical distance from O (resp., O') measures the share of aggregate skill in region a (resp., b). The shaded area in each diagram, denoted by M, represents the feasible domain of n_a and s_a. At each point on the upper (resp., lower) boundary of M the spatial configuration is such that all workers in region a are at least (resp., at most) as skilled as those in region b. The construction of these boundaries is as follows. Take any aggregate skill share s_a in region a. At the corresponding point on the upper boundary, region a has the smallest population share consistent with s_a. In other words, at each point on the upper boundary, there exists a *threshold skill level* z, such that all workers with skill level higher than z are in region a; those with skill level lower than z are in region b;

[17] Adopting a different nest between quantity and quality that still implies a positive relation between varieties' demand and their quality would lead to the same results as ours. The chosen Cobb–Douglas specification implies that vertical differentiation becomes more important in consumer tastes when vertical differentiation is more pronounced (σ is low). This specification makes the analysis tractable because it permits to have varieties' demand increasing linearly with quality, thus facilitating the aggregation of sales across firms.

[18] A detailed analysis under $c \geqslant \underline{s}$ is available from the authors upon request.

[19] For the ease of exposition, in Fig. 2, a continuous distribution of the population over the skill range S is assumed.

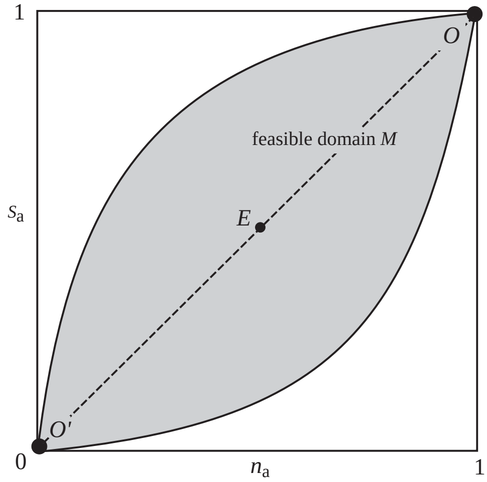

Fig. 2. Interpersonal distribution of skills and feasible interregional distribution of population and skill.

and those with skill level z may locate in either of the two regions. The lower is s_a, the higher the threshold skill level z: only a few of the most skilled workers are needed in region a to attain s_a. The lower boundary is obtained in a symmetrical way. In this case, for a given value of s_a, we must find the largest population share of region a consistent with s_a. At any interior point in M, workers are not completely sorted by skill levels, i.e., in each region we can find workers with both higher and lower skills than those of a given worker in the other region. [20]

The boundaries of area M have a straightforward interpretation: each of them is *the Lorenz curve of the interpersonal distribution of skill in the economy*. The size of area M corresponds to the value of the Gini index for the skill distribution. The larger is M, the greater the extent of inequality in the interpersonal distribution of skills. The standard core–periphery model is a particular case where M has size zero, and the feasible allocations are restricted on the 45° line which passes through the symmetric configuration at E ($n_a = s_a = 1/2$). Since n_a and s_a must necessarily be in area M, it follows also that the feasible regional allocations of population and skill are necessarily shaped by the extent of interpersonal inequality in the economy.

3. Temporary equilibrium

In this section, we derive the equilibrium conditions under a given regional distribution of workers. These are obviously the necessary conditions for a long-run equilibrium

[20] There may a continuum of workers' distributions, $f_a(\cdot)$, corresponding to each interior point in M.

in which workers have no incentive to relocate even if they can. In equilibrium, all workers must be choosing to work at the best available conditions. To derive such conditions, we first define a *temporary equilibrium*: a state of the economy in which the opportunities of workers are restricted within the region where they are "temporarily" located, i.e., *a temporary equilibrium is realized when*, *given the regional distribution of workers*, (i) *consumers maximize utility*; (ii) *firms maximize profits*; (iii) *profits are zero*; (iv) *all workers of the same skill attain the same utility level in their region*; (v) *all markets clear*. The economy is assumed to attain a temporary equilibrium instantaneously once the regional distribution of workers is given.

Note first that since intermediates are costlessly transportable, it must be that $\upsilon_a = \upsilon_b = \upsilon$. By choice of numeraire, we fix $\upsilon = 1$. Recall also that the final good is subject instead to transport costs of the iceberg type: for one unit to be shipped to the other region, $1-\tau$ units are lost in transit, and a fraction τ arrives to distant consumers, where $\tau \in (0,1)$.

Keeping this in mind, consider the profit maximization problem of a firm in region r. Since all firms share the same production technology and the price elasticity of demand is constant and equal to σ, all firms set a common mill price given by[21]

$$p = \sigma/(\sigma-1). \tag{6}$$

Thus, we omit henceforth the variety index. Moreover, we see that if product quality matters for firms' performance, it matters only in terms of quantities sold.

Expressions for total demand for a good with quality s in region r differ depending on whether the good is produced locally or in the alternative region. Denoting, respectively, by $X_r(s)$ and $X_r^*(s)$ the demand in region r for a local good and that for a product provided in the alternative region $-r$, from the consumer utility maximization we obtain

$$X_r(s) = s\,p^{-\sigma} I_r P_r^{\sigma-1}, \quad X_r^*(s) = (s-c)\tau^{\sigma-1} p^{-\sigma} I_r P_r^{\sigma-1}, \tag{7}$$

where I_r is the aggregate income in region r, given by[22]

$$I_r = 1 + \int_{s\in S} f_r(s) w_r(s)\,\mathrm{d}s, \tag{8}$$

and P_r is the price index in region r, given by

$$P_r \equiv p\hat{s}^{1/(1-\sigma)}[s_r + (s_{-r} - cn_{-r}/\hat{s})\tau^{\sigma-1}]^{1/(1-\sigma)}. \tag{9}$$

[21] As it is always the case with monopolistic competition, iceberg transport costs, and a CES–Dixit–Stiglitz representation of preferences, firms set the same markup over marginal costs in all locations (see, e.g., Chapter 4 in Fujita et al., 1999). It is important to bear in mind that when we speak about "tougher competition" in such a framework we refer to a downward shift in the individual demand curve at given price–cost margins.

[22] The first (resp., second) term in the RHS of (8) is the aggregate income of immobile (resp., mobile) workers in region r.

Given the fact that each unit of the final good requires one unit of intermediate input, the total amount of final goods produced in the economy at equilibrium equals the mass of unskilled labor, that is, two units. The market share of each seller depends on her skill level. The more skilled sellers are able to sell more as (7) indicates. Note also that transport and communication costs affect firms' sales to distant regions differently. We observe from (7) that transport costs affect the sales of worker-sellers of different skills in the same proportion, while communication costs fall proportionally more on the less skilled.

The equilibrium (operating) profit, $\pi_r(s)$, of a quality-s firm in region r can be easily obtained as a function of $X_r(s)$ and $X^*_{-r}(s)$. Recalling that the earnings of each skilled worker coincides with the firms's operating profit, and using (6), we obtain

$$\pi_r(s) = w_r(s) = \frac{1}{\sigma - 1}[X_r(s) + X^*_{-r}(s)]. \tag{10}$$

The wage rate is thus proportional to the quantity sold. Since by (7) the sales of the more skilled are less dependent on their local demand, their wage rate is also less dependent on their location. By solving (7)–(10) for $I_r P_r^{\sigma-1}$—which if multiplied by $p^{-\sigma}$ is the local demand for a firm offering a "standard" good (i.e., of quality 1)—we obtain

$$I_r P_r^{\sigma-1} = \frac{A_{-r} + B_r}{A_a A_b - B_a B_b}, \tag{11}$$

where $A_r \equiv P_r^{1-\sigma} - 1/\sigma\, p^{1-\sigma} s_r \hat{s}$ and $B_r \equiv 1/\sigma (p/\tau)^{1-\sigma}(s_r \hat{s} - c n_r)$. It can be shown that $A_r > B_r > 0$, which in turn implies $A_a A_b - B_a B_b > 0$. Thus, two types of local firm demand, $X_r(s)$ and $X_r^*(s)$, are determined uniquely and are strictly positive. It follows from (10) that the temporary equilibrium wage rate $w_r(s)$ is also determined uniquely and is strictly positive. The temporary equilibrium utility level of workers endowed with skill s and located in region r corresponds to their real wage, namely, their wage deflated by the locally prevailing cost of living index:

$$u_r(s) = w_r(s)/P_r. \tag{12}$$

4. Equilibrium

A temporary equilibrium is not necessarily an equilibrium of the economy in the "long run." Since skilled workers are freely mobile, they will search for the best opportunities available in the whole economy, moving across regions if profitable. Formally, workers are assumed to migrate toward the region that gives them a higher utility level. The adjustment process is assumed to be myopic and given by

$$\dot{f}_a(s) = \Phi(s|U,F) \quad \text{and} \quad \dot{f}_b(s) = -\Phi(s|U,F) \tag{13}$$

subject to (3), where $\dot{f}_r(s)$ is the time derivative of $f_r(s)$, $U \equiv \{u_a(s), u_b(s)\}_{s\in S}$, the set of temporary utility levels of each worker in each region, and $F \equiv \{f_a(s), f_b(s)\}_{s\in S}$ is the worker distribution between the two regions. We assume that the function $\Phi(\cdot|U,F)$ is a "smooth" functional of U and F, satisfying the following condition:[23]

$$\Phi(s|U,F) \quad \begin{cases} >0 & \text{if and only if } (u_a(s)-u_b(s))f_b(s)>0, \\ <0 & \text{if and only if } (u_b(s)-u_a(s))f_a(s)>0, \\ =0 & \text{otherwise}, \end{cases} \tag{14}$$

subject to (3).

The implication of condition (14) for the adjustment process (13) is obvious: the region where workers (regardless of skill levels) can attain higher utility is more attractive, under the constraint concerning the population distribution given by (3). The migration continues until no agent has an incentive to relocate.

The above considerations lead to the following definition of the (long-run) equilibrium: *an equilibrium is a state of the economy where* (i) *all the conditions for the temporary equilibrium are satisfied*; (ii) *all workers of the same skill attain the same utility level irrespective of their location.* So, an equilibrium is a temporary equilibrium that satisfies also the condition that $\dot{f}_r(s)=0$ for all s and r.

Now, how will the equilibrium distribution of workers across regions look like? In searching for equilibria we perform the following thought experiment. Starting from an arbitrary temporary equilibrium we ask whether a worker-seller of any skill level s located in region a has an incentive to relocate to region b.[24] She is willing to do so provided the utility level in region b is at least as high as what she is enjoying in region a. Denote by $u(s)$ *the relative standard of living* in region b (i.e., $u(s)\equiv u_b(s)/u_a(s)$) of a worker with skill level s. By applying (10)–(12), and using (7) and (11), we obtain the following expression:

$$u(s)=\frac{1}{P}\frac{sX+(s-c)\tau^{\sigma-1}}{s+(s-c)\tau^{\sigma-1}X}, \tag{15}$$

where P is *the relative cost of living index* in region b given by

$$P\equiv P_b/P_a=\left[\frac{s_a+(s_b-cn_b/\hat{s})\tau^{\sigma-1}}{s_b+(s_a-cn_a/\hat{s})\tau^{\sigma-1}}\right]^{1/(\sigma-1)} \tag{16}$$

and X represents *the firm relative local demand*, given by the ratio between the local demand for a firm selling a good of an arbitrary quality level s in region b and that in region a:

$$X\equiv\frac{X_b(s)}{X_a(s)}=\frac{A_a+B_b}{A_b+B_a}. \tag{17}$$

[23] In our setting, it can be verified that the temporary utility level of each skilled worker smoothly changes given a small change in the distribution of workers between the two regions. Here, we further assume that this smooth response of the utility levels in turn translates into that of migration rates. The concept of the smoothness of a functional is based on the so-called Volterra (1959) derivative.

[24] In the following, we use the terms, relative standard of living, relative utility level, and relative profitability, interchangeably.

A couple of remarks are in order. First, the firm relative local demand X is independent of s.[25] Thus, if the local demand for some variety with quality level s is larger in one region, then it is larger also for all the other varieties. Second, it is important to avoid misinterpretations concerning the meaning of X. If, say, $X > 1$, then an arbitrary firm has larger local sales if located in region b. This, however, does not mean that, on aggregate, sales, expenditure, and nominal income are greater in region b. Thus, it is important to bear in mind that X refers to the relative size of local markets from the viewpoint of a representative firm.

A region's attractiveness depends on the individual-specific skill level s, region-specific variables (X and P) and the level of transport and communication costs, τ and c. For workers of a given skill level, what matters are regional variables. Regional differences in market size and the cost of living index are shaped, in turn, by the allocation of population and skill across locations.[26]

A worker in region a of skill level s has a strict incentive to relocate if and only if $u(s) > 1$. The problem is perfectly symmetric when considering a worker with skill s who is instead temporarily located in region b. In equilibrium, we must have $f_a(s) > 0$ (resp., $f_b(s) > 0$) if and only if $u(s) \leqslant 1$ (resp., $u(s) \geqslant 1$).

5. Configurations, existence, and stability of equilibria

What equilibrium configurations are possible in our economy? Are there different ones? If so, under what conditions will each one be realized? Are these equilibrium configurations mutually exclusive or not? Are they stable? In this section we address these issues. First, we claim that in our setting, we have at most the following three equilibrium configurations:

Definition 1 (Equilibrium configurations). (i) Dispersed equilibrium: the most skilled worker in each region is more skilled than the least skilled worker in the other region; (ii) Concentrated equilibrium: all mobile workers locate in one region; (iii) Segmented equilibrium: the most skilled worker in one region is equally skilled or less skilled than the least skilled worker in the other region.

Dispersed equilibria include all the possible equilibria in which both regions have workers of each skill level. Graphically, they are located in the interior of the area delimited by the Lorenz curves (area M in Fig. 2). A special case of the dispersed equilibrium is the *symmetric equilibrium* (at point E in Fig. 2), where the aggregate skill and population are equal (though the skill distribution may differ) across regions. *Segmented equilibria* are located along the Lorenz curves. The *concentrated equilibrium* is at one of the two corners, O and O', in the figure, and is an extreme case

[25] Note that this result does not depend upon the Cobb–Douglas specification chosen to nest quality and quantity in preferences.

[26] Note that in the absence of communication costs, i.e., $c = 0$, the relative standard of living, the relative cost of living index, and the firm relative local demand is independent of the skill level of each agent, and only the aggregate skill and population in each region matter.

of the segmented equilibrium, where the threshold skill level is out of range $[\underline{s}, \bar{s}]$. Our model shares the possibility of the symmetric and concentrated equilibria with the standard core–periphery model. However, as we will see below, segmentation is the most typical stable equilibrium in our model.

In this section, we derive conditions for each equilibrium configuration defined in Definition 1. We also analyze the stability of equilibria.

5.1. Dispersed equilibria

Here, we show that it is impossible to have a stable equilibrium in the interior of the feasible domain M (Fig. 2). That is, the sorting mechanism mentioned in the introduction always pushes population-skill distribution toward the edge of the feasible domain, i.e., if there is any stable equilibrium, it should be either segmented or concentrated.

First, it can be shown that the firm relative local demand directly affects the relative profit and utility. Taking any $s \in (\underline{s}, \bar{s})$ we have by (15),

$$u(s) \geqslant u(s') \quad \text{if and only if } X \leqslant 1 \text{ for } s' < s. \tag{18}$$

How do we interpret these relations? Consider $X < 1$, so that region a offers a larger local market to each firm. Recall that a smaller value of $u(\cdot)$ makes region a more attractive. From (18) we see that region a is less attractive for high-skilled workers (or, if we prefer, for firms selling high-quality goods). This means that being located in a region offering a smaller local market for their goods is comparatively less disadvantageous for the more skilled. The explanation is simple. Communication costs fall more heavily on low-quality suppliers. Hence, those sellers that supply goods of higher quality are relatively footloose, since their sales can penetrate distant markets easily. We can reverse the argument for the case of $X > 1$. When $X = 1$, the relative profitability coincides for all firms.

Lemma 1. *Whenever local sales of any given firm differ across locations* (*i.e.*, $X \neq 1$) *the equilibrium can only be either concentrated or segmented. Conversely*, *in any dispersed equilibrium*, *we must have* $X = 1$.

Thus, a dispersed equilibrium requires equal local sales across locations, i.e., $X = 1$, which in turn implies that all workers must be indifferent between the two locations, i.e., $u(s) = 1$. It follows that *the regional allocation of workers is necessarily indeterminate in this case*. However, from (15), $X = 1$ and $u(s) = 1$ also imply that $P = 1$. Since the values of X and P depend upon how the aggregate skill and population are shared between the two regions, to characterize the dispersed equilibrium we must find out the particular class of worker distributions which are consistent with $X = P = 1$.

How does the regional shares of population and skill (n_a, s_a) affect the value of P and X? From (2) and (16), we have

$$P \geqslant 1 \text{ if and only if } n_a \geqslant (1 - \tau^{1-\sigma}) \frac{\hat{s}}{c} (s_a - 1/2) + 1/2. \tag{19}$$

The interpretation of (19) is straightforward. Given the regional allocation of aggregate skill (s_a), a larger number of sellers in a (n_a) lowers the cost of living there (P_a decreases relatively to P_b) due to consumers' love for variety. A larger number of local sellers is associated indeed with more varieties available locally. On the other hand, given the regional allocation of population, a larger aggregate skill in a region implies a greater average quality of goods, and hence, a smaller cost of living. It follows that the slope of the ($P=1$)-line is always negative in the (n_a, s_a)-space (refer to Fig. 3 in Section 5.2). The presence of communication costs softens the impact of the spatial distribution of skill on the relative cost of living: given a larger value of c, the relative price level more crucially depends on the size of locally available product variety rather than their quality.

Next, from (2) and (17) the impact of (n_a, s_a) on X is described as follows:

$$X \geqslant 1 \quad \text{if and only if } n_a \geqslant \left(1 - \frac{\sigma-1}{\sigma+1}\tau^{1-\sigma}\right)\frac{\hat{s}}{c}(s_a - 1/2) + 1/2. \tag{20}$$

This relation indicates first that given the regional allocation of skill (s_a), a larger number of competitors in a (n_a) will make the local firm demand smaller. When transport and/or communication costs are high, markets are segmented, and hence, the local firm demand is more sensitive to the variety size and average quality of goods in the own region.

From (19) and (20), it can be readily verified that

$$X = P = 1 \quad \text{if and only if } n_a = s_a = 1/2, \tag{21}$$

i.e., the ($P=1$)- and the ($X=1$)-lines never coincide except at the symmetric equilibrium (1/2,1/2). This proves that the symmetric equilibrium is the only dispersed equilibrium.

Is the symmetric equilibrium stable? The answer can be a positive one only if we can show that the symmetric equilibrium is restored after any possible perturbation to the distribution of workers. It is not difficult to show that this can never be the case. Consider a worker endowed with an arbitrary skill level s. From (15), we can show that the impact of the local market size and cost of living on the relative standard of living is described by the following condition:[27]

$$u(s) < 1 \quad \text{if} \quad \begin{cases} P \geqslant 1 \text{ and } X < 1, \text{ or} \\ P > 1 \text{ and } X \leqslant 1. \end{cases} \tag{22}$$

That is, given the same price level (resp., local firm market size), the region yielding a larger local market (resp., lower price level) is more attractive to any worker.

Now, consider a perturbation such that $s_a > 1/2$. Then, (19) and (20) imply that along the ($X = 1$)-line we must have $P > 1$, so that $u(s) < 1$ for all s by (22). This means that in any arbitrarily small neighborhood of the symmetric equilibrium there always exists a perturbation to the regional distribution of workers (along the ($X=1$)-line) which leads to a self-reinforcing agglomeration process towards region a. Symmetrically, we can always find a perturbation that induces $s_a < 1/2$ and that leads to a self-reinforcing agglomeration towards region b. Thus, we have the following proposition:

[27] A symmetric condition holds for the case $u(s) > 1$.

Proposition 1. *The symmetric equilibrium always exists, and is unstable. Moreover, it is the only dispersed equilibrium.*

In spite of the fact that the perfectly symmetric regional distribution of workers always induces an equilibrium, this equilibrium cannot be a stable one. It can always be found a perturbation to the symmetric configuration that leads to spatial inequality. The reason is that the workers at the top and those at the bottom of the skill ladder will seek for opposite locations. Being the former (resp., the latter) hit relatively less (resp., more) by the extent of local competition, they will move to the region where aggregate skill is higher (resp., lower), thus reinforcing the initial imposed asymmetry between locations. This "symmetry breaking" result is major point of departure of our analysis from the standard core–periphery model (and its existing variants), since there the symmetric equilibrium can instead be stable. The instability arising in our model is generated by the interaction of skill heterogeneity with communication costs.

It is to be noted that *the instability result persists even if the skill heterogeneity of workers is marginal, and as long as communication costs are positive.* This indicates that the simultaneous presence of worker heterogeneity and communication cost (i.e., non-proportionality between shipping costs and the value of shipped goods) is a necessary and sufficient condition for the instability result.

This observation leads to some remarks. The first concerns the robustness of the results arising under the conventional framework of the core–periphery model. Agent heterogeneity reveals a major source of instability, that may have so far received insufficient attention. Second, the presence of external economies related to human capital may not be necessary to predict the inevitability of regional differences in endowments and income. Pecuniary externalities are enough to do the job.

5.2. Concentrated and segmented equilibria

The concentrated equilibrium is another polar case that emerges in the standard core–periphery models, where all mobile workers concentrate in one region. In our model, however, the concentrated equilibrium can also be interpreted as a special case of the segmented equilibrium. In this particular case, although workers with different skills tend to have different location incentives, all worker-sellers prefer to be located in the same region.

In analyzing existence and stability of the concentrated equilibrium, it is convenient to answer first the following question: "Who are the workers that first leave the concentration when it ceases to be an equilibrium?" An easy argument shows that those workers that have the highest incentive to move away from a concentrated configuration are the least skilled. To see this, suppose all mobile agents are located in one region, say a. Then, the cost of living is higher in region b, where all goods must be transported from region a (i.e., $P > 1$). By (15) this means that region b must offer larger earnings to attract workers (i.e., $X > 1$). We know by (18) that in this case we must have $u(s) < u(s')$ for $s' < s$, namely, that relocating to region b is more advantageous for less skilled workers. It follows that if a worker has an incentive to move to region b, then all the workers with less skill want to do the same. Hence, we conclude that whenever the concentration breaks, the least skilled worker is the first one to relocate.

Next, under what condition is the concentration an equilibrium? As the discussion above suggests, the concentration breaks if the markets of the two regions are separated by sufficiently high transport costs or communication costs. Here, rather than completely characterizing the condition, we consider only the situation in which communication costs are sufficiently small so that they alone are not enough to break the concentration. This simplification is adequate, since the crucial assumption for our result is the non-proportionality between costs for shipping and the value of shipped goods. For this to occur it is sufficient that communication costs are strictly positive. In this restricted case, when all sellers are located in region a, they can easily penetrate the market in region b under zero transport cost (i.e., $\tau = 1$), so that no seller can get larger local demand in region b, i.e., $X < 1$. By (20), this is guaranteed if

$$\hat{\tau} \equiv \left(\frac{\sigma - 1}{1 + \sigma} \frac{1}{1 - c/\hat{s}} \right)^{1/(\sigma-1)} < 1. \tag{23}$$

Inequality (23) implies that this condition is equivalent to $\hat{s} > (\sigma + 1/2)c$. Under Assumption 1, it can be written as

Assumption 2. $(\sigma + 1/2)\underline{s} < \hat{s}$.

In the rest of the paper, we maintain this assumption. In this context, when transport costs are low, the concentration is an equilibrium. As transport costs increase, workers become less footloose, and seek the region with lower competition. As a result, the concentration tends to break. However, if the goods are more differentiated (σ is smaller), transport costs matter less, which in turn implies lower location dependence of income for all workers. In fact, if the goods are too differentiated ($\sigma < 2$), once all workers agglomerate in one region, no one would move away given any level of transport costs.[28]

The following proposition summarizes the results obtained [see Appendix A.1 for a proof of statement (ii)]:[29]

Proposition 2. (i) *When all skilled workers are concentrated in one location, if a worker with skill s' has an incentive to move to the other region, then all workers with $s \leqslant s'$ want to move as well.* (ii) *For $\sigma \leqslant 2$, the concentration is always an equilibrium, while for $\sigma > 2$, the concentration is an equilibrium if and only if $\tau^c \leqslant \tau \leqslant 1$, where $\tau^c \in (0, \hat{\tau})$ is the unique solution to the equation: $u(\underline{s}, \tau) = 1$.*

Notice that if the concentration is an equilibrium, it is by definition stable. Unlike the symmetric equilibrium, our model shares with the standard core–periphery models the basic ingredients of the equilibrium condition for the concentration. Provided that the degree of scale economies is not too high, the concentration is viable when the

[28] This corresponds to the so-called “black-hole” condition in the core–periphery model (see Section 4.6 in Fujita et al., 1999).

[29] Statement (i) of the proposition holds without Assumption 2.

transport costs are sufficiently low. In our setting, however, we need another condition for this result, namely, that the level of communication costs is not too high.

Concentrated equilibria are not the only type of stable outcome in our setting. From the results in the previous section, we know that segmented configurations, lying on the Lorenz curves as described in Fig. 2, are also possible candidates. Condition (18) implies that in a segmented equilibrium, less skilled workers tend to cluster in a region where their local sales are abundant but where locally supplied goods are of relatively low quality. Conversely, workers with sufficiently high skills find it convenient to locate where the local market for their good is smaller, but where locally provided goods are of relatively high quality. In summary,

Lemma 2. *In any segmented equilibrium, the local demand for each worker-seller in the high-skill region is smaller than that obtainable by being located in the low-skill region.*

Since for the less skilled it is more difficult to penetrate distant markets due to the loss of quality in transit, they are more tied to the local market. The more skilled are instead relatively footloose and are at ease in locating themselves where locally supplied goods are of high quality and the "true" cost of living index is lower. Given the smoothness of the migration process, any worker distribution (except those yielding a symmetric equilibrium) will inevitably converge to an equilibrium exhibiting regional inequality. In particular, we can show that a cycle is impossible in our economy.

For simplicity, we present the result under the assumption that $\sigma > 2$, so that the concentration is not always an equilibrium (Proposition 2(ii)) (for a proof, see Appendix A.2 in Mori and Turrini, 2000):

Proposition 3. (i) *If* $\hat{\tau} < \tau < 1$, *then the concentrated equilibria exist, and there is no segmented equilibrium.* (ii) *If* $\tau^c < \tau < \hat{\tau}$, *then the concentrated equilibria exist, and there may also exist one pair of segmented equilibria (one stable equilibrium and one unstable) with* $X < 1$ *and one pair with* $X > 1$. (iii) *If* $0 < \tau < \tau^c$, *then the concentrated configuration is not an equilibrium, and there exist at least one stable segmented equilibrium with* $X < 1$ *and one with* $X > 1$. (iv) *There is no cycle.*

Fig. 3 conveys the basic message of the proposition. Diagram (a) (resp., (b)) depicts case (i) (resp., (iii)), where the gray area is the same feasible domain M shown in Fig. 2.[30] The arrows in the diagrams indicate the typical direction of adjustments.

When transport costs are low (Diagram (a)), competition is "global", so that locating in the less populated region will not improve much sellers' market size. Thus, each worker-seller has little incentive to deviate from the larger region. The result is consistent with the core–periphery models. Namely, when transport costs are very low, full concentration is the only stable equilibrium.

[30] We omit the diagram for the case of intermediate transport costs ($\tau^c \leqslant \tau \leqslant \hat{\tau}$), since it adds no significant results. For this case, refer to Mori and Turrini (2000, Fig. 5(c,c'), Appendix A.2).

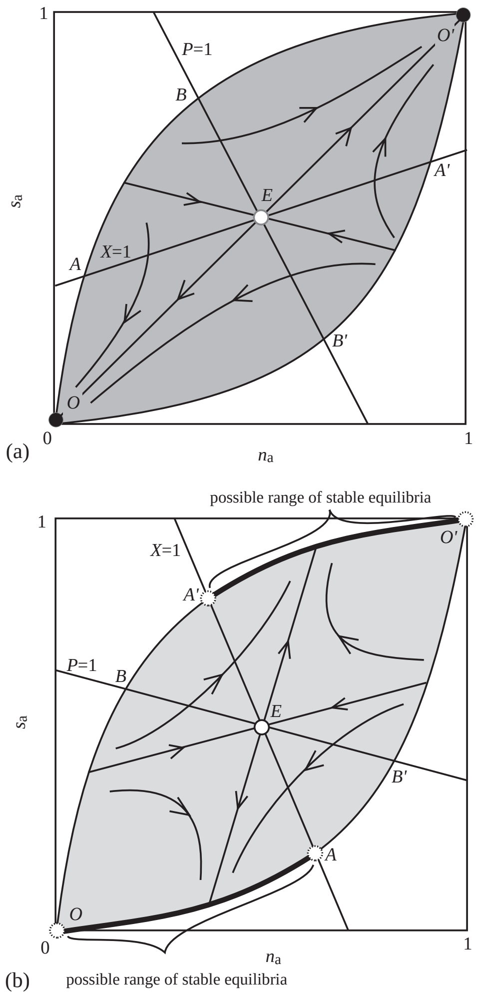

Fig. 3. Global dynamics of interregional population and skill distribution. (a) Low transport costs ($\hat{\tau} < \tau$). (b) High transport costs ($\tau < \tau^c$).

When transport costs are high (Diagram (b)), unlike the case of Diagram (a), the intensity of competition differs significantly across regions, especially when skill and population are relatively concentrated in one region. In particular, when all sellers are located in one region (O or O'), the least skilled would always want to deviate from the concentration to escape from tough competition (Proposition 2(i)). Since a segmented configuration is not an equilibrium along OB and $O'B'$ by Lemma 1, the only possible stable equilibria are the segmented ones along OA and $O'A'$.[31]

[31] The arrows in Diagram (b) are for simplicity drawn for the case in which there is a unique segmented equilibrium on OA and on $O'A'$.

In the case of intermediate transport costs ($\tau^c \leqslant \tau \leqslant \hat{\tau}$), the possible equilibrium configurations are either the same as those in Diagram (a), or a situation in between Diagrams (a) and (b) may happen, where both the concentrated and segmented equilibria coexist, as case (ii) in the proposition indicates.

6. Discussion

We saw that regional inequality is inevitable in our setting. This raises a number of questions. First, where should we expect the most skilled agents to be located? Our analysis above indicates that in any stable equilibrium, the aggregate skill in the region populated by the highly-skilled workers must be at least a half of the total, i.e., $\hat{s}/2$. This can be seen from the fact that the share of aggregate skill in region a at point A'—the intersection of the $(X = 1)$-line and the upper Lorenz curve—can never be smaller than 1/2 by (20). As for the population share in this high-skill region, things are less clear-cut. Fig. 3(b) shows that in all segmented equilibria the region inhabited by the high skilled must have a minimum population size (i.e., when the skilled go to region a, n_a is necessarily bigger than the value found at the intersection A' of the $(X=1)$-line and the upper Lorenz curve). This minimum population size, however, could be below 1/2, provided the $(X = 1)$-line is negatively sloped (i.e., transport costs are high).[32]

We can say the following. The most skilled are always found in wealthier locations, where aggregate human capital, and aggregate income (and welfare) is higher.[33] Wealthier locations may end up being less populated, because these are inhabited by the "privileged few" who are skilled and rich.

How can we explain that the highly-skilled are to be found in wealthier locations? The reason is tied to the different impact of communication costs on sellers of different skills. Since the more skilled are always closer to the global market (less dependent upon local markets), they are not as sensitive as the less skilled to being located where their local individual demand is low. In other words, the local firm demand is small in rich thriving areas. Fierce competition is the price to pay for being located where the best are: the local total demand must be shared among the top sellers, and the share accruing to each is necessarily small. However, note that the cost of living index is lower in the high-skill region than the low-skill region, since a wider range of high-quality goods are available there. This low price index compensates for the disadvantages of a limited local market, yielding a high standard of living in the high-skill region.[34] We summarize these remarks in the next corollary of Proposition 3.

[32] Recall that all the segmented equilibria on the upper and lower Lorenz curves are, respectively, along the $O'A'$ and along OA segments in Fig. 3(b).

[33] Denote by $I = I_b/I_a$ relative nominal incomes. After some algebra it is shown that $I > 1 \Leftrightarrow s_a < (1 - c/\hat{s})/(2 - c/\hat{s}) + (c/\hat{s})/(2 - c/\hat{s})n_a$. It is easily checked that this condition is satisfied if and only if the most skilled are located in b.

[34] A straightforward corollary of this result is that immobile workers are necessarily better-off in the high-skill location. Their nominal income is the same in both locations, but the real income is higher in the high-skill region because of a lower cost-of-living index.

Corollary 1. (i) *In any stable equilibrium, one region has higher aggregate skill and income, as well as a higher average utility level than the other.* (ii) *Workers at the top of the skill ladder are always located where aggregate skill is higher.*

A second issue we would like to discuss is the following. What is the role of transport and communication costs in shaping the spatial distribution of economic activities? Distance matters in our model because of both transport and communications costs. As for transport costs, we see from Proposition 3 that whenever transport costs are sufficiently low (i.e., τ approaches 1) the equilibrium configuration is necessarily concentrated. The maximum extent of agglomeration occurs. This is a basic result of the standard core–periphery model. As transport costs rise, however, we cannot hope for a symmetric configuration in our model as in the case of the standard core–periphery model. The configuration will be segmented, with one high-skill region and one with low skills. We can say more. If transport costs are not too high, the high-skill region will be the one with the larger population. This is easily understood recalling that, by (20), the ($X = 1$)-line is positively sloped if $\tau > ((\sigma - 1)/(\sigma + 1))^{1/(\sigma-1)}$ (refer to Fig. 3(a)). By the previous arguments, any segmented equilibrium must exhibit larger population in the high-skill region in this case. Recall also that the ($X = 1$)-line rotates clockwise as transport costs fall. So, we see that a reduction in transport costs still tends to raise the extent of agglomeration as in the standard core–periphery model. As τ becomes close to 1, most of the mobile agents must be located in the high-skill region.

What about communication costs? It is possible to show that a fall of communication costs may have very different effects on the equilibrium configuration depending on the level of transport costs.

Consider the case where transport costs are low ($\tau > ((\sigma-1)/(\sigma+1))^{1/(\sigma-1)}$). Then, communication costs are dominating transport costs in the overall cost for transit. In this case, after a reduction in communication costs, even low-skilled worker-sellers become footloose. We see that in this context the agglomeration of human capital is necessarily associated with that of population. This can be seen from the fact that the range of possible segmented equilibria (OA and $O'A'$ on the Lorenz curves) shrinks after a fall in communication costs (refer to (20)). On the other hand, if transport costs are high ($\tau < ((\sigma - 1)/(\sigma + 1))^{1/(\sigma-1)}$), through a symmetric mechanism, even highly-skilled workers are tied to the local market, and seek the region with less competition. As a result, the agglomeration of human capital is not likely to be associated with that of population as communication costs fall. These findings are summarized in the next corollary to Proposition 3:

Corollary 2. (i) *When transport costs are low* ($\tau > ((\sigma-1)/(\sigma+1))^{1/(\sigma-1)}$) *the region with higher aggregate skill has also a larger population.* (ii) *When* $\tau > ((\sigma-1)/(\sigma+1))^{1/(\sigma-1)}$ (*resp.*, $\tau < ((\sigma - 1)/(\sigma + 1))^{1/(\sigma-1)}$), *the minimum share of population in the high-skill region rises* (*resp.*, *falls*) *as communication costs fall.*

The above results may shed light on some real-world phenomena. It may help to explain for instance the empirical fact that high concentrations of human capital have been associated with those of population in recent decades characterized by substantially decreasing communication costs and a fairly low level of transport costs (Glaeser et al., 1992; Black and Henderson, 1998). The model also helps in qualifying predictions concerning the impact of reduced communication costs on spatial agglomeration. In general, the analysis gives a further argument in favor of those that are skeptical about an inevitable dispersion of economic activities as a result of lower costs for communications: segmentation or concentration will remain the only stable outcome even with very small communication costs.[35] However, communication costs may matter for the extent of agglomeration because they affect the relation between the equilibrium location of skills and population.

7. Concluding remarks

Economic agents are not all equal. This is a basic fact of life. Some workers, sellers, entrepreneurs are more skillful, brilliant, or simply luckier than others. In a world of unequal abilities and fortunes, we necessarily observe rich and poor places: wealth and human capital are not evenly represented across towns, regions or states. This paper addressed these points formally. Our economy is populated by sellers that differ in their skills, thus performing differently in the marketplace. We have shown that modifying the well-known core–periphery model of location in such a way that agent heterogeneity is allowed has major analytical implications. The sustainability of a symmetric location pattern, which is a common feature of the existing "new economic geography" models, breaks. Regional inequality is inevitable. Most skilled agents are attracted by wealthier locations, those where human capital and wealth are more abundant.

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[35] See Glaeser (1998) for a discussion of the tendencies in urban agglomeration in the face of falling communication costs. See also Gaspar and Glaeser (1998) for an argument in favor of greater need of face-to-face contact, and agent proximity in the presence of lower communication costs.

Appendix A

A.1. Proof of Proposition 2(ii)

Without loss of generality, assume all workers are located in region a ($s_a = n_a = 1; s_b = n_b = 0$). In this case, the expressions for the relative price index (16) and the firm relative local demand (17) boil down to $P=(1-c/\hat{s})^{1/(1-\sigma)}/\tau$ and $X=(\sigma-1)/(\sigma+1)\tau^{1-\sigma}/(1-c/\hat{s})$, respectively. By substituting these expressions into (15), we obtain

$$u(s)=(1-c/\hat{s})^{1/(\sigma-1)}\frac{(\sigma-1)\tau^{2-\sigma}+(\sigma+1)(1-c/s)(1-c/\hat{s})\tau^{\sigma}}{(\sigma+1)(1-c/\hat{s})+(\sigma-1)(1-c/s)}. \tag{24}$$

From the investigation of (24), we can show the following. If $\sigma \leqslant 2$, then $u(\cdot)=0$ at $\tau=0$, and $u(\cdot)$ is increasing in $\tau>0$. If $\sigma>2$, then $u(\cdot)$ approaches ∞ as τ approaches 0, and as τ increases from 0, it first decreases, then increases (in particular, it is convex for $\tau>0$). Statement (ii) follows from this result together with statement (i). Q.E.D. □

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