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*Olena Ivus**
*Alireza Naghavi***
*Larry D. Qiu****

* Queen's University
** University of Bologna, Johns Hopkins University and Centro Studi Luca d'Agliano
*** University of Hong Kong

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Olena Ivus Alireza Naghavi Larry D. Qiu*

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Abstract

This paper develops a North-South trade model with heterogeneous labour and horizontally differentiated products and compares the implications of two policies: Southern intellectual property rights (IPRs) and Northern immigration policy that aims to attract Southern talent as means of preempting imitation. Individuals self-select into becoming entrepreneurs and innovate (imitate) in the North (South). The likelihood of imitation depends on product quality, imitator's ability, and strength of IPRs. Several interrelated channels of competition are identified. Allowing high-ability migration when IPRs protection in the South is weak shifts imitation to low-quality and innovation to high-quality products. The outcome is in stark contrast to the policy of strengthening IPRs, which limits low-quality imitation and encourages low-quality innovation. High-ability migration also increases the income of low-ability entrepreneurs, as well as the average quality of products in the high-ability imitation sector in the South.

Keywords: Intellectual property rights; High-skilled migration; Imitation; Innovation; Product quality; Entrepreneur ability

JEL classification: F22, O31, O34, J24, K37, O38.

*Olena Ivus: Smith School of Business, Queen's University, olena.ivus@queensu.ca. Alireza Naghavi: Department of Economics, University of Bologna; Johns Hopkins University SAIS Europe; Centro Studi Luca d'Agliano, alireza.naghavi@unibo.it. Larry D. Qiu: Faculty of Business and Economics, University of Hong Kong, larryqiu@hku.hk. We thank seminar participants at Kyiv School of Economics (2019), Kobe University (2019), Kyoto University (2019), and participants at International Conference on "Time Zones, Offshoring, Economic Growth and Dynamics" (Kobe, 2019), Asia Pacific Trade Seminars (Tokyo, 2019), and Workshop on "Migration, Globalization and the Knowledge Economy" (Utrecht, 2019). Olena Ivus acknowledges financial support from the Social Sciences and Humanities Research Council of Canada (SSHRC) Insight Grant "Intellectual property rights, innovator migration and technology diffusion," and Larry D. Qiu acknowledges financial support from Hong Kong SAR Government's RGC Competitive Earmarked Research Grant 2017-2020, No. HKU 17502217.

1 Introduction

While the advanced economies (the North) may view imitation as a threat to innovation activity, imitation remains vital to technological progress in a large number of non-industrialized developing economies (the South). To limit imitation, the North has been actively pushing to raise the global standards of intellectual property rights (IPRs), while the South has firmly resisted this push, fearing that the reforms would limit its access to Northern innovative products and technologies and thereby, stifle its development. Meanwhile, the South is concerned with an alarming loss of talent ("brain drain") as high-ability individuals migrate to the North, where intellectual assets have stronger protection and career prospects are better. Figure 1 shows, using data from Miguelez and Fink (2013) and Park (2008), that inventor emigration rates are highest among countries with weak IPRs. This negative association persists over time, even as the strength of IPRs continues to rise around the world to meet global standards. However, as the talent pool declines in the South, so does the imitation threat. Perhaps rather than pushing for a strengthening of IPRs in the South, the North can use its immigration policy to preempt imitation? Industry leaders in the North have long emphasized the importance of attracting high-ability individuals from the South, arguing that a more open high-skilled migration policy would allow the Northern firms to limit competitive pressure from the Southern firms and also, strengthen their innovation capacity in the process. In countries where the enforcement of IPRs continues to pose a serious concern, the competitive pressure from imitators is of utmost importance for Northern firms.

[Figure 1 here]

This paper explores the role of migration policy as a tool to battle imitation. The key questions in this respect are: What are the consequences of global talent flows? How do the two policies—i.e., strengthening Southern IPRs and opening the North to high-ability immigration—compare in their impact on imitation activity in the South and innovation activity in the North? We introduce an occupational choice model of an innovative North and an imitative South with two dimensions of heterogeneity: product varieties spread over a continuum of quality, and individuals in each region differing in their entrepreneurial ability. This framework allows us to study the impact of migration and IPRs policies on the composition of innovated and imitated products, while accounting for competition between innovated varieties in the North, between imported innovated and domestic imitated varieties in the South, and also between imitators with different abilities in the South.

An individual in each region can become an entrepreneur or a production worker. Entrepreneurs earn rents associated with their specific ability. In the North, a higher ability entrepreneur innovates a higher quality variety and earns more rents. In the South, entrepreneurs imitate innovated varieties: imitation is easier for high-ability entrepreneurs but for a given ability, higher quality varieties are more difficult to imitate. Accordingly, the imitator's expected rents rise with product quality initially but fall eventually as the likelihood of successful imitation falls. The endogenous entrepreneurship decisions determine the effects of policies on innovation and imitation.

We first consider the policy of strengthening IPRs in the South. Stronger IPRs reduce the expected rents of imitators and so, some low-ability entrepreneurs exit imitation. Thus, the policy restrains imitation but only of low-quality varieties. In the North, the competitive pressure on entrepreneurs falls, the innovators' rents rise, and low-ability individuals enter innovation. More innovative products are introduced as a result, but these innovations are of low quality. Such nominal effects on low-quality products put into question the effectiveness of Southern IPRs protection in limiting imitation and encouraging innovation.

We next examine the policy of allowing a number of high-ability Southern entrepreneurs to migrate to the North. High-ability migrants exit high-quality imitation in the South and enter high-quality innovation in the North, thereby creating three effects. First, the mass of high-ability entrepreneurs falls in the South and high-quality imitation set contracts. This is the direct "brain drain" effect. Second is the negative competition effect. The mass of high-quality (innovated and imitated) varieties available for consumption rises, and consumers in each region lower their spending on each individual variety. The entrepreneurial rents fall in each region as a result, discouraging low-quality imitation and innovation. But the competitive pressure on low-ability entrepreneurs in the South also falls as high-quality imitation contracts and Northern innovation shifts away from low-quality products. The rents of low-ability Southern entrepreneurs thus rise, encouraging low-quality imitation. This third effect – the positive competition effect – is particularly strong in the "less-developed" South, where IPRs protection is weak and wage rate is low.

The two policies differ critically in their impact on the entrepreneurial activity in each region. Opening the North to high-ability immigration from the "less-developed" South shifts imitation from high- to low-quality products (as high-quality imitation set contracts and low-quality imitation set expands). This result is in sharp contrast to the policy of strengthening Southern IPRs, which limits low-quality imitation. Moreover, while high-ability migration shifts innovation from low- to high-quality products (as low-quality innovation set contracts and the mass of high-quality innovated varieties rises), strengthening IPRs in the South promotes low-quality innovation.

Further, both policies serve to decrease the aggregate income in the South and increase the aggregate income in the North but the distribution of income within each region is affected differently. With high-ability migration from the "less-developed" South, the aggregate income in the production sector falls in the South and rises in the North. But a strengthening of IPRs in the South has the opposite effect: the aggregate production income rises in the South and falls in the North. These results suggest that openness of the North to high-ability immigration is more appealing from a global development perspective as it helps the "less-developed" South reduce its reliance on the production sector and promotes transition towards a more entrepreneurial economy.

Last, we compare the policies' impacts on average product quality. These results are more nuanced. Nonetheless, under certain conditions, we find that allowing for high-ability migration from the "less-developed" South increases average product quality in the innovation sector in the North and also, in the high-ability imitation sector in the South. The latter effect could be viewed as a compensation for the "brain drain". A strengthening of IPRs in the South, by contrast, decreases the average quality in the innovation sector and increases the average quality in the low-ability imitation sector, but does not affect the average quality in the high-ability imitation sector.

The underlying purpose of IPRs is to encourage innovation by protecting innovators' intellectual assets from imitation. The literature on trade-related IPRs has uncovered the important role of IPRs protection in stimulating technology transfer via international trade and multinational firm activity (see, e.g., Helpman, 1993; Maskus and Penubarti, 1995; Lai, 1998; Branstetter et al., 2006; Branstetter et al., 2007; Ivus, 2010, 2015; Canals and Sener, 2016; Ivus et al., 2017; Ivus and Park, 2019; Ivus and Saggi (forthcoming)). Migration literature has also uncovered the important role of skilled immigration in promoting innovation in host countries by offering expertise and entrepreneurial skills (Ganguli et al., 2020).[1] Bosetti et al. (2015) argues that policies aimed at attracting skilled migrants could boost innovation in Europe. Miguelez and Moreno (2015) investigates the effectiveness of European policies aimed at attracting foreign researchers and examines preconditions under which migrant researchers help foster EU competitiveness in innovation. Stuen et al. (2012) finds that international doctoral students contribute to knowledge production at scientific laboratories in the US, and argues that visa restrictions on the entry of high-quality students are particularly costly for academic innovation. Kerr and Lincoln (2010) shows that total science and engineering employment and invention increases with more H-1B visa admissions in the US, mainly due to immigrants' direct contributions. More recently, Akcigit et al. (2017) and Morrison et al. (2018) find, using historical data, that immigrant inventors to the US substantially contribute to innovation in the country and generate positive knowledge spillovers that benefit the productivity of US inventors.

The two strands of literature (on trade-related IPRs and on high-skilled international migration) have been pursued in isolation of each other. Few studies to date have studied IPRs and migration in a unified framework. Mondal and Gupta (2008) introduced migration into the Helpman (1993) model of trade-related IPRs and showed that a strengthening of IPRs in the South decreases the share of imitated products, shifts labour from the South to the North, and promotes innovation.

[1] Also, innovator mobility is one channel through which technologies are diffused worldwide (e.g., Hunt and Gauthier-Loiselle, 2010; Moser et al., 2014). Ganguli (2015), for example, draws upon the influx of Russian scientists to the US after the end of the Soviet Union to show how high-skilled immigrants contribute to knowledge diffusion as a basis for innovation and economic growth.

In Kuhn and McAusland (2009), skilled emigrants improve the quality of goods to the benefit of the source country consumers; the increment in quality is large when the source and host countries have more disparate IPRs. McAusland and Kuhn (2011) considered whether governments looking to attract migrant innovators have a greater incentive to protect intellectual property. The paper shows that while advanced developing countries pass overreaching IPRs (i.e., relative to globally efficient levels), poorer developing countries with large innovator emigration find it optimal to under-protect IPRs, assuming goods produced abroad are valued less at home. In Naghavi and Strozzi (2015), IPRs protection works as a moderating factor between migration and innovation. Knowledge acquired by emigrants flows back to the source country through diaspora networks, and this flow-back generates brain gain when the source country's IPRs are strong.

An important question in the study of high-skilled migration flows is: What type of entrepreneurial activities would the migrants have undertaken in their origin countries had they not had the opportunity to emigrate? The majority of innovative activity is concentrated in high-income economies; while in many low-income countries, entrepreneurs engage in imitative research and development activities because economic institutions, regulatory environment, infrastructure, and market and business sophistication are not conducive to innovation (Dutta et al., 2016).[2]

And yet, despite the rich array of evidence on the relationship between high-skilled immigration and innovation, there has been practically no discussion of imitation in the international migration literature. One notable exception is Kerr (2008), which outlines a leader-follower model of technology transfer through ethnic networks and then empirically evaluates the role of U.S. ethnic scientific and entrepreneurial communities for international technology transfer to the countries of origin. While the model developed in our paper is novel in that the occupational decisions of entrepreneurs are endogenously determined in the presence of labour and product market heterogeneity, the model in Kerr (2008) is similar to ours in three respects. First, the focus in Kerr (2008) is on the steady-state equilibrium where the leading economy does not imitate and the follower's economy does not innovate. Similarly, our framework is of innovative North and imitative South. Kerr (2008) argues that such equilibrium is not limited to extremely poor regions but may also arise in emerging economies with a small inventive stock.[3] Secondly, similar to our paper, Kerr (2008) assumes that entrepreneurial scientists imitate foreign inventions for domestic use only, and a larger stock of foreign inventions provides a larger pool of technologies available for imitation. Last, Kerr (2008)'s assumption that the follower's expatriates work only in the research sector is

[2] Talent and IPRs protection are not sufficient for a developing South to transition to an innovating economy. The developing South must also possess a critical level of complementary research, technological, and marketing assets to ensure that its entrepreneurs can absorb the transferred technologies, introduce new products, and exploit innovations.

[3] In Kerr (2008), the equilibrium is determined by the researcher's imitation-versus-invention decisions: the follower's entrepreneurial scientists choose to engage in imitation when their productivity for invention is less than their productivity for imitating foreign technologies.

also akin to ours.

This paper highlights the importance of studying skilled migration through the framework of entrepreneurial decisions and in this respect, contributes to two recent important approaches in the literature. The first approach is to analyze skilled migration from the perspective of the firm. Kerr et al. (2015a) argues that firms are mostly absent from the literature on the impact of skilled migration, and Kerr et al. (2015b) and Kerr et al. (2016) underscore the need for greater clarity in understanding the heterogeneity in firm employment choices and the impact of migration on reallocation across firms and aggregate productivity.[4] The second approach is to consider the impact of skilled migration on labour-market outcomes when knowledge and skills are specialized. Borjas and Doran (2012) examine the productivity effects of the influx of Soviet mathematicians after 1992 on the American (and global) mathematics community. The paper finds that the influx into the U.S. created competitive pressure in both the job market and in the market for codified knowledge; consequently, marginal U.S. mathematicians became much more likely to move to lower quality institutions and exit knowledge production altogether. In Peri and Sparber (2011), immigration affects occupations and the associated skill content of native-born employees. The paper finds that immigrants with graduate degrees specialize in occupations demanding quantitative and analytical skills, while native-born employees with the same educational attainment move to occupations requiring interactive and communication skills. Wadhwa et al. (2012) find that during 2006-2012, immigrants founded 24% of engineering and technology companies in the U.S. and 44% of high-tech companies in Silicon Valley.[5] In line with this literature, our paper emphasizes that the heterogeneous ability of entrepreneurs and the diverse range of quality among products available for imitation are important considerations when studying the impacts of high-ability migration from the developing South. The labor and product market heterogeneity affects the type of activities entrepreneurs engage in, as well as their productivity, and also determines the degree of competitive pressure that high-ability immigration creates in labor and product markets.

The paper proceeds as follows. Section 2 describes the basic North-South model of trade with full enforcement of IPRs in the South. Section 3 focuses on the imitation and innovation decisions when the IPRs in the South are partially enforced, describes the model equilibrium, and examines the impact of strengthening IPRs. In Section 4, we assume the North introduces

[4]Kerr et al. (2015a) stresses that many skilled immigrant admissions are driven by firms themselves (e.g., the H-1B visa). In our paper, we assume that the number of migrants is exogenously given by the Northern immigration quota. We abstract from the migration decisions and instead, focus on the heterogeneity in the entrepreneurial decisions. Immigration creates competitive pressure and affects the occupational choice and productivity of native entrepreneurs in the North and the remaining entrepreneurs in the South.

[5]Kahn et al. (2017) finds that the documented higher rates of entrepreneurship among immigrants (an immigrant entrepreneurship premium) are specific to science-based entrepreneurship, which is consistent with immigrants having larger endowments of entrepreneurial skills or greater alertness to entrepreneurial opportunities. Kerr and Kerr (2016) quantifies immigrant contributions to new firm creation, and finds that immigrant-founded businesses have faster employment grow than native businesses, particular in high-tech sectors.

a migration quota for the entry of high-ability individuals and study how this policy impacts the model equilibrium. Section 5 compares the policies' impacts on the entrepreneurial activity, the composition of innovated and imitated varieties, and income and welfare. Section 6 concludes.

2 The Basic Set-up

Suppose the world consists of two regions: imitative South ($i = S$) and innovative North ($i = N$). The North invents new products because of its comparative advantage in research and development (R&D). The South imitates products because the regulation and enforcement of IPRs is not so strict as to make imitation prohibitively expensive. The mass of individuals is L_N in the North and L_S in the South. An individual can be an entrepreneur or a production worker in each region. An entrepreneur is an innovator in the North and an imitator in the South. Each worker provides one unit of labour, and labour is the only factor of production.

There exists a continuum of differentiated product varieties, denoted by $\upsilon \in \Upsilon$ and a homogeneous good υ_0, which is treated as the numeraire. The instantaneous utility function of the representative agent in region $i = \{S, N\}$ is given by

$$U_i = c_i(\upsilon_0) + \left[\int_{\Upsilon} z(\upsilon)^{1-\theta} c_i(\upsilon)^{\theta} d\upsilon \right]^{\frac{1}{\theta}},$$

where $c_i(\upsilon_0)$ is the consumption of the homogeneous good; $z(\upsilon)$ and $c_i(\upsilon)$ respectively denote the quality and consumption level of variety υ; and $\theta = (\sigma - 1)/\sigma$, with $\sigma > 1$ being the constant elasticity of substitution in consumption.[6]

2.1 A Closed Northern Economy

Individuals in the North are spread over a continuum of ability $a \in [0, 1]$, distributed with density $g_N(a)$ and cumulative distribution $G_N(a)$. The North innovates and produces differentiated product varieties, the market for which is monopolistically competitive with free entry and exit into each variety. An individual can become an entrepreneur who innovates a variety υ of quality $z(\upsilon)$. High ability leads to high-quality variety and thus, ability a and quality z are the same. Consequently, the density of entrepreneurs that innovate varieties of quality $z(\upsilon)$ is given by $f(z) = L_N g_N(z)$, with cumulative distribution denoted by $F(z)$.

Individuals who do not become entrepreneurs become production workers.[7] Workers are homogeneous in their production ability. One unit of labour produces one unit of the innovated variety

[6] Product quality is modelled as a utility shifter in, for example, Hummels and Klenow (2005) and Hallak (2006).
[7] We assume no imitation in the North. Thus, no one becomes an imitator.

or w (> 1) units of the homogeneous good. The homogeneous good market is competitive. We normalize the price of the homogeneous good to one. Thus, the North's wage rate is w.[8]

Assume for now that the Northern economy is closed. The representative individual in the North faces the following budget constraint: $Y_N = c_N(\upsilon_0) + E_N$, where Y_N denotes total income that is equal to total expenditure; $E_N = \int_\Upsilon p_N(\upsilon)c_N(\upsilon)d\upsilon$ is the aggregate expenditure on the differentiated varieties that is exogenously given; and $p_N(\upsilon)$ and $c_N(\upsilon)$ are the price and consumption of variety υ. Maximizing U_N subject to Y_N yields the following "quality-adjusted" demand for each differentiated variety in the North:

$$c_N(\upsilon) = z(\upsilon)\left[\frac{p_N(\upsilon)}{P_N}\right]^{-\sigma}\left(\frac{E_N}{P_N}\right), \tag{1}$$

where $P_N \equiv \left[\int_\Upsilon z(\upsilon)p_N(\upsilon)^{1-\sigma}d\upsilon\right]^{1/(1-\sigma)}$ is the price-quality index. Assuming an interior solution, consumption of the homogeneous good is determined by the residual income $c_N(\upsilon_0) = Y_N - E_N$.[9]

An entrepreneur that innovates variety υ enjoys a monopoly in that variety because varieties of the same quality are horizontally differentiated. The entrepreneur charges the monopoly price $p_N(\upsilon) = p_N = w/\theta$, which equals a fixed markup above marginal cost w, and sells $c_N(\upsilon)$. This decision is the same for all varieties of the same quality z. The entrepreneurial rents are given by

$$\pi(z) = (p_N - w)c_N(z) = \frac{z}{\sigma}\left(\frac{p_N}{P_N}\right)^{1-\sigma}E_N. \tag{2}$$

The rents rise with product quality: $\pi'(z) > 0$. With free entry into innovation, there must exist a cutoff $\hat{z}$ such that the entrepreneurs with quality (ability) $z = \hat{z}$ are indifferent between working as production workers or innovators: $\pi(\hat{z}) = w$. Entrepreneurs with $z > \hat{z}$ earn rents $\pi(z) > w$. From (2), the implicit solution for the cutoff $\hat{z}$ is

$$G(\hat{z}) \equiv \hat{z}E_N - \sigma w \int_{\hat{z}}^{1} zdF(z) = 0. \tag{3}$$

Thus when Northern economy is closed, the quality set of innovated varieties is $\Phi = [\hat{z}, 1]$. We assume that E_N is large enough such that $G(1/2) > 0$. Then, $\hat{z} < 1/2$ and the set Φ is wide.

[8] In each region, the homogeneous good sector works as a buffer to absorb all residual labour.

[9] With quasi-linear preferences and a given aggregate expenditure on the differentiated varieties, prices determine the demand for the differentiated varieties and the residual income determines the homogeneous good demand. The consumption of the homogeneous good adjusts to fully absorb a change in income; there is no income effect on the consumption of differentiated varieties. Alternatively, one could fix the income share of the differentiated varieties. This specification would embed the income effect on the consumption of differentiated varieties, as it would imply that the aggregate expenditure on differentiated varieties adjusts in a fixed proportion to the change in income.

2.2 Open Economy with Full IPRs Enforcement in the South

We consider the global economy with free trade. In the South, individuals become either production workers or entrepreneurs who imitate Northern varieties. One unit of labour produces one unit of the imitated differentiated variety or one unit of the homogeneous good.[10] The homogeneous good is traded at the price of $p(\upsilon_0) = 1$. Thus, the Southern wage rate is equal to one.

The likelihood of a Northern variety being imitated depends on the strength of IPRs protection in the South. We measure the strength of IPRs in the South by $\Omega \in [0, 1]$, which is the probability that a patent is enforced by the Southern government.[11] In the North, IPRs are fully enforced.

In this subsection, we consider the full enforcement of IPRs in the South: $\Omega = 1$. In this case, the Northern entrepreneurs can sell their varieties in the global market without risking imitation, and all individuals in the South produce the homogeneous good.

Let $c_X(\upsilon)$ denote the demand for variety υ in the South and Υ^F denote the set of innovated varieties when $\Omega = 1$. The representative individual in the South faces this budget constraint: $Y_S = c_S(\upsilon_0) + E_S$, where Y_S denotes total income that is equal to total expenditure; $c_S(\upsilon_0)$ is the consumption of the homogeneous good; $E_S = \int_{\Upsilon^F} p_X(\upsilon) c_X(\upsilon) d\upsilon$ is the aggregate expenditure on the imported varieties; and $p_X(\upsilon)$ and $c_X(\upsilon)$ are the price and consumption of variety υ. Maximizing U_S subject to Y_S, we obtain the South's "quality-adjusted" demand for each variety:

$$c_X(\upsilon) = z(\upsilon) \left[\frac{p_X(\upsilon)}{P_X} \right]^{-\sigma} \left(\frac{E_S}{P_X} \right),$$

where $P_X \equiv \left[\int_{\Upsilon^F} z(\upsilon) p_X(\upsilon)^{1-\sigma} d\upsilon \right]^{1/(1-\sigma)}$ is the price-quality index. Assuming an interior solution, consumption of the homogeneous good is equal to $c_S(\upsilon_0) = Y_S - E_S$.

A Northern entrepreneur that innovates variety υ sets the monopoly price $p_X(\upsilon) = p_N(\upsilon) = w/\theta$ in both regions and earns the global rents given by

$$\pi_N(z) = (p_N - w)[c_N(z) + c_X(z)] = \frac{z}{\sigma} \left(\frac{p_N}{P_N} \right)^{1-\sigma} (E_N + E_S), \tag{4}$$

where $P_N = P_X$. As $\pi_N'(z) > 0$ and entry into innovation is free, there exists a cutoff $\bar{z}$ such that entrepreneurs with quality $z = \bar{z}$ are indifferent between working as production workers or innovators: $\pi_N(\bar{z}) = w$. Using (4), we find the following implicit solution for the cutoff $\bar{z}$:

$$G^F(\bar{z}) \equiv \bar{z} (E_N + E_S) - \sigma w \int_{\bar{z}}^{1} z dF(z) = 0. \tag{5}$$

[10] Labour in the North is more productive in the production of homogeneous goods than labour in the South. Labour productivity in differentiated goods is the same in both regions.

[11] This assumption is similar to that of Grossman and Lai (2004).

Thus in an open global economy with full IPRs enforcement in the South, the quality set of innovated varieties is $\Phi^F = [\bar{z}, 1]$.

Based on the above analysis, we establish the following result:

Proposition 1. *A unique equilibrium* $\hat{z} \in (0, 1)$ *exists in a closed Northern economy, such that individuals with ability* $a \geq \hat{z}$ *choose to become entrepreneurs innovating varieties of quality* $z \geq \hat{z}$ *and individuals with ability* $a < \hat{z}$ *choose to become production workers. This equilibrium is instead* $\bar{z} \in (0, \hat{z})$ *in an open global economy with full IPRs enforcement in the South.*

Proof: See Appendix.

Since $\bar{z} < \hat{z}$, trade enlarges the quality set of innovated varieties.

3 Imitation

In this section, we consider partial enforcement of IPRs in the South: $\Omega < 1$. A Northern entrepreneur who sells an innovated variety of quality z in the global market enjoys a monopoly in the South until this variety is imitated. An imitated variety is a copy of the Northern variety that infringes on Northern patents; hence while it is sold to consumers in the South, it is not exported to the North, where IPRs are fully enforced.

Each individual in the South can enter imitation and become an entrepreneur to compete with imports of the Northern variety that it imitates. Imitators have the same production technology as innovators but lower marginal cost (labour cost). We assume that competition is on the basis of price. According to Grossman and Helpman (1993), in the event of imitation, there are two possible pricing scenarios, depending on the size of the North-South marginal cost differential. First is the narrow-gap case, when $w < 1/\theta$. In this case, a Southern entrepreneur enjoys a relatively small cost advantage over its Northern rival and is unable to set the monopoly price without fear of being undercut. Thus, the Southern entrepreneur sets the price $p_S = w$ and earns the rents $\pi_S(z) = (w - 1)c_S(z)$. Second is the wide-gap case, when $w > 1/\theta$. A large marginal cost differential allows the Southern entrepreneur to charge the monopoly price $p_S = 1/\theta$ and earn the rents $\pi_S(z) = (1/\theta - 1)c_S(z)$. We consider the wide-gap case.

The quality set of innovated varieties is $\Phi^F = [\bar{z}, 1]$ in an open global economy with full IPRs enforcement in the South ($\Omega = 1$) and $\Phi = [\hat{z}, 1]$ in the closed Northern economy case. In an open global economy with partial IPRs enforcement in the South ($0 < \Omega < 1$), the quality set of innovated varieties will include all varieties with quality $z \geq \hat{z}$ and possibly some varieties with quality $z \in (\bar{z}, \hat{z})$. We let Φ^P denote this set and determine it below.

Let Υ^P be the set of innovated varieties and $\Upsilon^M \subset \Upsilon^P$ be the set of imitated varieties when $\Omega \in (0, 1)$. Then, the representative agent in the South faces the following budget constraint: $Y_S = c_S(\upsilon_0) + E_S$, where the aggregate expenditure on the differentiated varieties is $E_S = \int_{\Upsilon^P \setminus \Upsilon^M} p_X(\upsilon)c_X(\upsilon)d\upsilon + \int_{\Upsilon^M} p_S(\upsilon)c_S(\upsilon)d\upsilon$. That is, E_S is the sum of the expenditure on the imported varieties, which are priced at $p_X(\upsilon)$, and that on the imitated varieties, which are priced at $p_S(\upsilon)$. Maximizing U_S subject to Y_S, we obtain the South's "quality-adjusted" demand for each innovated and imitated variety, respectively:

$$c_X(\upsilon) = z(\upsilon)\left[\frac{p_X(\upsilon)}{P_S}\right]^{-\sigma}\left(\frac{E_S}{P_S}\right); \qquad c_S(\upsilon) = z(\upsilon)\left[\frac{p_S(\upsilon)}{P_S}\right]^{-\sigma}\left(\frac{E_S}{P_S}\right), \tag{6}$$

where $P_S \equiv [\int_{\Upsilon^P \setminus \Upsilon^M} z(\upsilon)p_X(\upsilon)^{1-\sigma}d\upsilon + \int_{\Upsilon^M} z(\upsilon)p_S(\upsilon)^{1-\sigma}d\upsilon]^{1/(1-\sigma)}$ is the price-quality index. Assuming an interior solution, consumption of the homogeneous good is determined by the residual income $c_S(\upsilon_0) = Y_S - E_S$.

Since the two markets are segmented, the consumption decision in the North is the same as that in Section 2.1, except that the set of innovated varieties is now Υ^P rather than Υ.

Next, we need to determine the quality set of innovated varieties Φ^P and the quality set of imitated varieties Φ^M when $\Omega \in (0,1)$. To simplify our exposition, we first examine the imitation decision based on the assumption that all varieties with $z \geq \bar{z}$ are available for imitation (which arises when $\Omega = 1$). We then consider the innovation decision.

All Southern individuals have the same ability in production but there are two types of imitation ability: low a_L and high a_H, with their respective density given by g_S^L and g_S^H. We assume that all high-ability (H) individuals become entrepreneurs and engage in imitation.[12] Then, the mass of H entrepreneurs is given by $f_S^H = L_S g_S^H$. The occupational choice of low-ability (L) individuals is not certain: g_S^L is large enough that some of the L individuals become production workers while others become imitators. Thus the mass of L entrepreneurs is $f_S^L < L_S g_S^L$. We determine f_S^L below.

Let the likelihood of the innovated variety with quality z being imitated by $j \in \{L, H\}$ entrepreneur be defined as follows:

$$\mu^j(z) \equiv (1-\Omega)m^j(z), \tag{7}$$

where $dm^j(z)/dz < 0$ and $m^L(z) < m^H(z) < 1$. Higher-quality varieties are more difficult to imitate but for a given quality z, imitation is easier for high-ability entrepreneurs. A useful approach is to adopt the following linear specification for the imitation rate $m^j(z)$:

$$m^j(z) = \begin{cases} \alpha_{j1} - \beta_{j1}z & \text{for } z \leq a_j, \\ \alpha_{j2} - \beta_{j2}z & \text{for } z > a_j. \end{cases} \tag{8}$$

To ensure that $m^L(z) < m^H(z) < 1$ for any z, we restrict the parameters as follows: (i) $0 < \alpha_{L1} < \alpha_{L2} < \alpha_{H1} < \alpha_{H2} < 1$, (ii) $0 < \beta_{L1} < \beta_{L2} < \beta_{H1} < \beta_{H2}$, and (iii) $\alpha_{L2} - \beta_{L2} < \alpha_{L1} - \beta_{L1} <$

[12] This is the equilibrium outcome under the condition (11).

$\alpha_{H2} - \beta_{H2} < \alpha_{H1} - \beta_{H1}$. Figure 2 shows that $m^H(z)$ and $m^L(z)$ are linear in z with a kink at α_H and α_L, respectively.

[Figure 2 here]

Two cases could arise in equilibrium: (i) the L and H entrepreneurs do not compete for imitating the same variety, i.e., the L and H imitation sets do not overlap, and (ii) the L and H entrepreneurs compete for a range of varieties, i.e., the L and H imitation sets overlap. In the non-overlapping case, the expected rents of a Southern j imitator of a variety with quality z, are as follows:

$$\pi_S^j(z) = \mu^j(z)\,(p_S - 1)\,c_S(z) = \frac{z\mu^j(z)}{\sigma}\left(\frac{p_S}{P_S}\right)^{1-\sigma} E_S. \tag{9}$$

A Southern entrepreneur earns a larger amount of rents from imitating a high-quality variety, but the likelihood of successfully imitating a high-quality variety is low. One can reasonably assume that the latter effect is particularly strong in the range of high-quality varieties $z > a_j$ so that the rents $\pi_S^j(z)$ depend on z as follows: $d\pi_S^j(z)/dz \geq 0$ for $z \leq a_j$ and $d\pi_S^j(z)/dz < 0$ for $z > a_j$. Then the entrepreneurs prefer to imitate high-quality varieties when $z \leq a_j$ and low-quality varieties when $z > a_j$.[13] Lemma 1 follows:

Lemma 1. *It is true that $d\pi_S^j(z)/dz \geq 0$ for $z \leq a_j$ and $d\pi_S^j(z)/dz < 0$ for $z > a_j$ under the following sufficient condition:*

$$\frac{\alpha_{j1}}{\beta_{j1} + \beta_{j2}} < a_j \leq \frac{\alpha_{j1}}{2\beta_{j1}}. \tag{10}$$

This condition requires that the line $m^j(z)$ is sufficiently flat when $z \leq a_j$ and sufficiently steep when $z > a_j$. Figure 3 shows the imitators' rent functions in relation to the imitation rate functions when the condition (10) holds (and all varieties with $z \geq \bar{z}$ are available for imitation).

[Figure 3 here]

All Southern entrepreneurs of the same type are *ex-ante* identical. We assume the following allocation mechanism. If a sufficient number of Southern entrepreneurs imitates varieties of quality z (within Φ^P), then the entrepreneurs will be randomly allocated such that the density of Southern entrepreneurs imitating the varieties is equal to the density of Northern entrepreneurs innovating the varieties, that is, $f(z)$. For the H entrepreneurs, the best varieties to imitate are those with $z = a_H$ and by continuity, the second best varieties are those with quality, for example z'_H, right next to a_H (on each side). The mechanism will first randomly select $f(a_H)$ of H entrepreneurs

[13] The assumption requires that $\alpha_{j1} - 2\beta_{j1}z \geq 0$ for all $z \leq a_j$ and $\alpha_{j2} - 2\beta_{j2}z < 0$ for all $z > a_j$. This is true under the condition (10), since $\alpha_{j1} - \beta_{j1}a_j = \alpha_{j2} - \beta_{j2}a_j$ or $\alpha_{j2} = \alpha_{j1} + (\beta_{j2} - \beta_{j1})a_j$ from (8).

to imitate varieties with $z = a_H$, and then, will randomly select $f(z'_H)$ to imitate z'_H. This will go on until all H entrepreneurs are allocated. The quality set of varieties imitated by the H entrepreneurs is continuous and therefore, we can denote it as $[z_{H0}, z_{H1}]$, with $a_H \in (z_{H0}, z_{H1})$. In equilibrium, the mass of Southern entrepreneurs must equal the mass of Northern entrepreneurs over the range $[z_{H0}, z_{H1}]$, which requires the following:

$$L_S g_S^H = \int_{z_{H0}}^{z_{H1}} dF(z). \tag{11}$$

A low-ability individual enters imitation as long as rents $\pi_S^L(z)$ exceed the Southern wage rate. There is a pecking order for imitation, with $z = a_L$ being the best quality to imitate. As with the H entrepreneurs, the mechanism will randomly allocate the L entrepreneurs to each variety, starting with $z = a_L$. We can denote the quality set of varieties imitated by the L entrepreneurs as $[z_{L0}, z_{L1}]$, with $a_L \in (z_{L0}, z_{L1})$. With free entry into imitation, the equilibrium requires that $\pi_S^L(z) \geq 1$ for any $z \in [z_{L0}, z_{L1}]$, where the rents at the two end points are equal to the Southern wage rate. Then the mass of L entrepreneurs is endogenously determined as follows: $f_S^L = \int_{z_{L0}}^{z_{L1}} dF(z)$.

Thus, the quality set of imitated varieties is $\Phi^M = [z_{L0}, z_{L1}] \cup [z_{H0}, z_{H1}]$. To simplify our analysis, we impose the following two restrictions: $z_{L0} \in (\hat{z}, a_L)$ and $z_{H1} < 1$.[14] Lemma 2 provides a sufficient condition for these two restrictions to hold.

Lemma 2. *Assume that* $\alpha_{L1}/\beta_{L1} > 1$ *and* $\int_{a_H}^{1} dF(z) > f_S^H$. *Then,* $z_{L0} \in (\hat{z}, a_L)$ *and* $z_{H1} < 1$.
Proof: See Appendix.

Whether the sets $[z_{L0}, z_{L1}]$ and $[z_{H0}, z_{H1}]$ overlap or not depends on a_L, a_H, g_S^H, and the parameters of the imitation rate function. Lemma 3 establishes a sufficient condition for the non-overlapping case.

Lemma 3. *In equilibrium, the two quality sets of imitated varieties,* $[z_{L0}, z_{L1}]$ *and* $[z_{H0}, z_{H1}]$, *do not overlap, i.e.,* $z_{L1} < z_{H0}$, *if* a_L *is sufficiently small and* a_H *is sufficiently large.*
Proof: See Appendix.

We show in the Appendix that a sufficient condition for no overlap is $\tilde{z}_{L1} < \tilde{z}_{H0}$, where $\tilde{z}_{L1}$ and $\tilde{z}_{H0}$ are respectively implicitly defined by $(\alpha_{L2} - \beta_{L2}\tilde{z}_{L1})\tilde{z}_{L1} - (\alpha_{L1} - \beta_{L1}\hat{z})\hat{z} = 0$ and $(\alpha_{H1} - \beta_{H1}\tilde{z}_{H0})\tilde{z}_{H0} - (\alpha_{H2} - \beta_{H2}) = 0$. This condition requires a low a_L and a high a_H, so that the gap between the best varieties to imitate is large. The non-overlapping case accentuates the

[14] We rule out the possibility that $z_{H1} = 1$ to avoid the corner solution where z_{H0} is determined by $\int_{z_{H0}}^{1} dF(z) = f_S^H$ and $\pi_S^H(z_{H0}) > \pi_S^H(1)$.

heterogeneity in entrepreneurial ability, which is the focus of our paper. Thus in what follows, we study the non-overlapping case.[15] From Lemma 3 and its proof, we obtain Lemma 4.

Lemma 4. *In the non-overlapping case,* $\Phi^M = [z_{L0}, z_{L1}] \cup [z_{H0}, z_{H1}]$, $z_{L1} < z_{H0}$, *the end points defining the two intervals,* z_{j0} *and* z_{j1}, *are determined by (11), and*

$$\begin{aligned} \pi_S^L(z_{L0}) = \pi_S^L(z_{L1}) = 1, \\ \pi_S^H(z_{H0}) = \pi_S^H(z_{H1}) > 1. \end{aligned} \tag{12}$$

Figure 4 shows one equilibrium in the non-overlapping case. The expected rents of a j imitator of a variety of quality $z \in \Phi^M$ are given by[16]

$$\pi_S^j(z) = \frac{z\mu^j(z)}{\sigma}\left[\frac{w^{\sigma-1}E_S}{\psi_N + (w^{\sigma-1}-1)\psi_S}\right], \tag{13}$$

$$\text{where} \quad \psi_N \equiv \int_{\bar{z}}^1 z dF(z) \quad \text{and} \quad \psi_S \equiv \sum_j \int_{z_{j0}}^{z_{j1}} z\mu^j(z) dF(z). \tag{14}$$

[Figure 4 here]

■ Innovation

So far, we have assumed that all varieties with $z \geq \bar{z}$ are available for imitation. Next, we determine the quality set of innovated varieties Φ^P. Proposition 2 follows.

Proposition 2. *Suppose that IPRs are partially enforced in the South. Then, there exists a unique equilibrium* $z^P \in (\bar{z}, \hat{z})$, *such that individuals with ability* $a \geq z^P$ *choose to become entrepreneurs innovating varieties of quality* $z \geq z^P$ *and individuals with ability* $a < z^P$ *choose to become production workers. The quality set of innovated varieties is* $\Phi^P = [z^P, 1]$.

Proof: See Appendix.

The cutoff z^P is implicitly defined by

$$G^P(z^P) \equiv z^P(E_N + \xi E_S) - \sigma w \int_{z^P}^1 z dF(z) w = 0, \tag{15}$$

where ψ_N is now defined as follows: $\psi_N \equiv \int_{z^P}^1 z dF(z)$, and $\xi \equiv \psi_N / [\psi_N + (w^{\sigma-1} - 1)\psi_S] < 1$.

[15] For completeness, we provide the solution for the overlapping case in the Appendix.

[16] This result follows from (9), where $P_S = \left[\psi_N p_N^{1-\sigma} + \psi_S(p_S^{1-\sigma} - p_N^{1-\sigma})\right]^{1/(1-\sigma)}$, $p_N = w/\theta$, and $p_S = 1/\theta$.

The innovator $z \in \Phi^P \setminus \Phi^M$ does not risk imitation and earns the global rents given by:[17]

$$\pi_N(z) = \frac{z}{\sigma}\left[\frac{E_N}{\psi_N} + \frac{E_S}{\psi_N + (w^{\sigma-1} - 1)\psi_S}\right]. \tag{16}$$

The innovator $z \in \Phi^M$ is priced out of the Southern market by the j imitator with probability $\mu^j(z)$ and so, earns the expected global rents given by:

$$\pi_N(z) = \frac{z}{\sigma}\left\{\frac{E_N}{\psi_N} + \frac{[1 - \mu^j(z)]E_S}{\psi_N + (w^{\sigma-1} - 1)\psi_S}\right\}. \tag{17}$$

It follows from (16) that $\pi'_N(z) > 0$ for z in the set $\Phi^P \setminus \Phi^M$, which is a union of the three continuous sets: $[z^P, z_{L0})$, (z_{L1}, z_{H0}), and $(z_{H1}, 1]$. Also from (17), $\pi'_N(z) > 0$ for z in the set Φ^M, which is a union of the two continuous sets: $[z_{L0}, z_{L1}]$ and $[z_{H0}, z_{H1}]$. The competition with imitators reduces the innovators' rents in the Southern market but does not affect the rents in the North. Thus, $\pi_N(z) > w$ for any $z > \hat{z}$, where $\hat{z}$ is the closed Northern economy cut-off, such that $\hat{z} < z_{L0}$. Hence, the cutoff z^P must be below $\hat{z}$. The cutoff z^P must also be above $\bar{z}$, which is the open economy cutoff when IPRs are fully enforced in the South. Innovators with $z \in (\bar{z}, \hat{z}]$ do not risk imitation but a partial enforcement of IPRs in the South still reduces their rents, because it enables competitive pressure from imitated varieties (that are priced relatively low).[18]

We now have a complete model and can assess the effects of different policies on the entrepreneurial activity in each region.[19]

■ Strengthening IPRs in the South

Stronger IPRs in the South reduce the likelihood of the innovated variety of quality z being imitated: $d\mu^j(z)/d\Omega = -m^j(z) < 0$. The j imitator's expected rents $\pi^j_S(z)$ thus fall, and the quality sets of imitated and innovated varieties adjust in response. Proposition 3 follows.

Proposition 3. *Strengthening of IPRs in the South decreases z^P (the innovation set expands), increases z_{L0} and decreases z_{L1} (the L imitation set contracts), and has no effect on z_{H0} and z_{H1}.*
Proof: See Appendix.

Figure 5 illustrates the effects of stronger IPRs. The direct effect is given by a proportionate shift in $\pi^j_S(z)$. As the L imitators' rents $\pi^L_S(z)$ fall, the cutoff z_{L0} rises and the cutoff z_{L1} falls. The L entrepreneurs to the immediate right of z_{L0} and left of z_{L1} exit imitation when their rents fall

[17] This condition follows from $\pi_N(z) = (p_N - w)[c_N(z) + c_X(z)]$ and the demand functions given in (1) and (6), where $P_N^{1-\sigma} = \psi_N p_N^{1-\sigma}$, $P_S^{1-\sigma} = \psi_N p_N^{1-\sigma} + \psi_S(p_S^{1-\sigma} - p_N^{1-\sigma})$, $p_N = p_X = w/\theta$, and $p_S = 1/\theta$.

[18] We note that $\xi < 1$ in the equation (16) when $\Omega < 1$.

[19] We detail on the composition of labour and income in the two regions in the Appendix.

below the Southern wage rate. A reduction in the H imitators' rents $\pi_S^H(z)$, by contrast, does not affect the cutoffs z_{H0} and z_{H1}, as long as $\pi_S^H(z_{H0}) = \pi_S^H(z_{H1}) > 1$ as in Lemma 4.

[Figure 5 here]

When the L imitation set contracts, the imitated varieties are replaced with the imported innovated varieties at the margin. The competitive pressure on the Northern innovators thus falls and their rents $\pi_N(z)$ rise.[20] Northern individuals with ability to the immediate left of z^P enter innovation, and the innovation set expands into lower-quality varieties (z^P falls). The mass of innovated varieties available for consumption rises and so, consumers in the South lower their spending on each imitated variety, causing the imitators' rents $\pi_S^j(z)$ to further fall.[21] This negative *competition* effect of stronger IPRs exacerbates the contraction in the L imitation set.

4 Migration

Suppose now that the North introduces a migration quota M for the entry of high-ability individuals from the South. The quota is restrictive: $M < g_S^H L_S$, and high-ability individuals are randomly selected to fill the quota. The high-ability individuals who migrate become innovators in the North, introducing varieties with quality a_H. With zero fixed cost of migration, a high-ability individual will migrate as long as their entrepreneurial rents rise following migration. This requires that $\pi_N(a_H) > \pi_S^H(a_H)$, which holds under the following sufficient condition:[22]

$$\frac{E_N}{E_S} > w^{\sigma-1}. \tag{18}$$

When E_N/E_S is high enough, the migration quota is 100% filled. After migration, the mass of H entrepreneurs in the South falls while the mass of entrepreneurs in the North rises by M and so, the condition (11) changes to the following:

$$g_S^H L_S - M = \int_{z_{H0}}^{z_{H1}} dF(z) + M. \tag{19}$$

Opening the North to high-ability migration increases the mass of innovated varieties of quality a_H available for consumption and also, for imitation. The conditions (12) and (16), which together

[20] Since the imitated varieties are priced relatively low, the price-quality index $P_S = \left[\psi_N p_N^{1-\sigma} + \psi_S (p_S^{1-\sigma} - p_N^{1-\sigma})\right]^{1/(1-\sigma)}$ rises as ψ_S falls, holding ψ_N constant.

[21] The price-quality index $P_S = \left[\psi_N p_N^{1-\sigma} + \psi_S (p_S^{1-\sigma} - p_N^{1-\sigma})\right]^{1/(1-\sigma)}$ falls as ψ_N rises, holding ψ_S constant.

[22] Using (13) and (17), we rewrite $\pi_N(a_H) > \pi_S^H(a_H)$ as $E_N/(\xi E_S) > (w^{\sigma-1}+1)\mu^H(a_H) - 1$. The condition (18) follows because $\xi < 1$ and $\mu^H(a_H) < 1$.

with (19) define the equilibrium cutoffs, are unchanged, but the quality-adjusted number of innovated varieties and the quality-adjusted expected number of imitated varieties are now respectively given by:

$$\psi_N \equiv \int_{z^P}^{1} z dF(z) + a_H M \quad \text{and} \quad \psi_S \equiv \sum_j \int_{z_{j0}}^{z_{j1}} z\mu^j(z) dF(z) + a_H \mu^H(a_H) M. \tag{20}$$

The North's migration policy has two effects on the H imitation activity. First is the direct "brain drain" effect. The mass of H entrepreneurs falls in the South while the mass of a_H varieties available for imitation rises. The high-quality imitation set contracts in response, as z_{H0} rises and z_{H1} falls to ensure that $\pi_S^H(z_{H0}) = \pi_S^H(z_{H1})$ and the condition (11) holds. Second is the negative competition effect. The mass of a_H (innovated and imitated) varieties available for consumption in the South rises and as Southern consumers lower their spending on each individual variety, the competitive pressure on the H imitators rises. The price-quality index $P_S = \left[\psi_N p_N^{1-\sigma} + \psi_S (p_S^{1-\sigma} - p_N^{1-\sigma})\right]^{1/(1-\sigma)}$ falls (due to "love of variety"), and the H rents $\pi_S^H(z)$ thus also fall. Nonetheless, a reduction in $\pi_S^H(z)$ *per se* does not affect z_{H0} and z_{H1}, as long as $\pi_S^H(z) > 1$. Thus overall, the H imitation set unambiguously contracts.

Unlike the H imitation cutoffs, the L imitation cutoffs z_{L0} and z_{L1} respond to a change in the L imitators' rents $\pi_S^L(z)$, because $\pi_S^L(z_{L0}) = \pi_S^L(z_{L1}) = 1$. But the impact on $\pi_S^L(z)$ is ambiguous and depends on three forces at play. First is the negative competition effect: $\pi_S^L(z)$ falls as the competitive pressure from the a_H (innovated and imitated) varieties rises. Second is the positive competition effect caused by a contraction in the H imitation set. When high-quality imitated varieties are replaced with more expensive imported innovated varieties at the H margins, the price-quality index $P_S = \left[\psi_N p_N^{1-\sigma} + \psi_S (p_S^{1-\sigma} - p_N^{1-\sigma})\right]^{1/(1-\sigma)}$ rises and so, the L imitators' rents $\pi_S^L(z)$ also rise. Last is the positive competition effect due to a contraction in the innovation set. As we discuss below, high-ability migration causes the cutoff z^P to rise. As the innovation set contracts, the competitive pressure on the L imitators falls and so, the rents $\pi_S^L(z)$ rise.

Importantly, the overall impact of high-ability migration on the L imitation activity depends on the strength of IPRs in the South, as measured by Ω. We show in the Appendix (proof of Proposition 4) that the value of Ω influences the intensity of competitive pressure from the high-quality imitated varieties as determined by the following function:

$$\Lambda(\Omega) = (w^{\sigma-1} - 1)[2z_{H0}\mu^H(z_{H0}) - a_H \mu^H(a_H)] - a_H. \tag{21}$$

This function determines the relative strength of the first two forces above (i.e., the negative competition effect caused by an increase in the mass of a_H varieties and the positive competition effect caused by a contraction in the H imitation set). When $\Lambda(\Omega) > 0$, the second force dominates and so, the competitive pressure on the L imitation activity falls overall. The L imitation set expands (as z_{L0} falls and z_{L1} rises) in this case. But when $\Lambda(\Omega) < 0$, the strength of the third force (i.e., the positive competition effect due to a contraction in the innovation set) also matters. We find that the negative competition effect dominates overall, so that the L imitation set contracts, when $\Lambda(\Omega) < \Lambda^P(\Omega)$, where the function $\Lambda^P(\Omega)$ depends on the elasticity of ψ_N with respect to z^P and the relative aggregate expenditure on the differentiated varieties in the South as follows:

$$\Lambda^P(\Omega) = -\frac{a_H \varepsilon^P}{1 + \varepsilon^P + \xi E_S/E_N} < 0, \quad \text{where} \quad \varepsilon^P \equiv -\frac{d\psi_N}{dz^P}\frac{z^P}{\psi_N}. \tag{22}$$

Figure 6a plots $\Lambda(\Omega)$ and $\Lambda^P(\Omega)$ under the condition that $\Lambda(0) \geq 0$ or equivalently that:

$$(w^{\sigma-1} - 1)[2z_{H0}m^H(z_{H0}) - a_H m^H(a_H)] \geq a_H. \tag{23}$$

The condition (23) requires a low Southern wage rate (a high w).[23] Under this condition, the second force (the positive competition effect caused by a contraction in the H imitation set) is at least as strong as the first force (the negative competition effect caused by an increase in the mass of a_H varieties) when $\Omega = 0$ but as Ω rises, the second force loses its relative strength so that $d\Lambda(\Omega)/d\Omega < 0$. As Ω rises from zero to one, $\Lambda(\Omega)$ falls at a constant rate from $\Lambda(0) \geq 0$ to $\Lambda(1) = -a_H$ while $\Lambda^P(\Omega)$ rises from $\Lambda^P(0) < 0$ to $\Lambda^P(1) > -a_H$. The two functions intersect at a single point, $\Omega = \bar{\Omega}$. The L imitation set expands when $\Omega < \bar{\Omega}$ and contracts when $\Omega > \bar{\Omega}$.

[Figure 6 here]

Suppose instead that $\Lambda(0) \leq \Lambda^P(0) < 0$, for which the following condition is sufficient:[24]

$$(w^{\sigma-1} - 1)[2z_{H0}m^H(z_{H0}) - a_H m^H(a_H)] < a_H/2, \tag{24}$$

The condition (24) requires a high Southern wage rate (a low w).[25] Under this condition, the first force (the negative competition effect caused by an increase in the mass of a_H varieties) dominates over the entire range of Ω, so that $\Lambda(\Omega) < \Lambda^P(\Omega) < 0$.[26] Figure 6b plots $\Lambda(\Omega)$ and $\Lambda^P(\Omega)$ under the assumption that $2z_{H0}m^H(z_{H0}) - a_H m^H(a_H) < 0$, in which case the condition (24) holds and $d\Lambda(\Omega)/d\Omega < 0$. As Ω rises from zero to one, $\Lambda^P(\Omega)$ rises from $\Lambda^P(0) > \Lambda(0)$ to $\Lambda^P(1) > \Lambda(1) = -a_H$, while $\Lambda(\Omega)$ falls at a constant rate. The two functions do not intersect, and the L imitation set contracts for any Ω.

[23] The following is sufficient for (23) to hold: $(w^{\sigma-1}-1)[2\tilde{z}_{H0}m^H(\tilde{z}_{H0}) - a_H m^H(a_H)] - a_H \geq 0$, where $\tilde{z}_{H0} < z_{H0}$ and from the proof of Lemma 3, $\tilde{z}_{H0}$ is implicitly defined by $(\alpha_{H1} - \beta_{H1}\tilde{z}_{H0})\tilde{z}_{H0} - (\alpha_{H2} - \beta_{H2}) = 0$.

[24] This is because $\Lambda^P(0) > -a_H/2$ from (22).

[25] This condition holds if $2z_{H0}m^H(z_{H0}) - a_H m^H(a_H) < 0$ (for which $a_L > a_H/2$ is sufficient) or if $(w^{\sigma-1} - 1)m^H(a_H) \leq 1/2$.

[26] This is because $\Lambda(\Omega)$ is linear in Ω and $\Lambda^P(1) > \Lambda(1) = -a_H$.

With the preceding discussion, we establish the following proposition:

Proposition 4 (Migration and imitation). *Allowing high-ability migration into the North contracts the H imitation set (z_{H0} rises and z_{H1} falls) and affects the L imitation set as follows:*
(i) Suppose the Southern wage rate is high enough that the condition (23) holds. Then there exists a unique critical value of the strength of IPRs protection in the South, given by $\bar{\Omega}$, such that the L imitation set expands (z_{L0} falls and z_{L1} rises) if $\Omega < \bar{\Omega}$, and the L imitation set contracts (z_{L0} rises and z_{L1} falls) if $\Omega > \bar{\Omega}$.
(ii) Suppose the Southern wage rate is low enough that the condition (24) holds. Then, the L imitation set contracts (z_{L0} rises and z_{L1} falls).

Proof: See Appendix.

With regard to innovation, the impact on the cutoff z^P is not immediately clear because migration changes the competitive environment for the innovators in both markets. In the Northern market, migration creates the negative competition effect: the innovators' rents $\pi_N(z)$ fall as the mass of the a_H innovated varieties rises. In the Southern market, the competitive pressure from the a_H (innovated and imitated) varieties also rises but this negative competition effect is opposed by the positive competition effect caused by a contraction in the H imitation set. The competition pressure from the L imitated varieties changes as well: it falls when the L imitation set contracts and rises otherwise. We find that the cutoff z^P rises if $\Lambda(\Omega) < \Lambda^L(\Omega)$, where[27]

$$\Lambda^L(\Omega) = \frac{a_H E_N}{\xi^2 E_S}\left[1 + (1-\xi)\left(\frac{\varepsilon_{L1}}{\epsilon_{L1}} + \frac{\varepsilon_{L0}}{\epsilon_{L0}}\right)\right], \tag{25}$$

$$\varepsilon_{L1} \equiv \frac{d\psi_S}{dz_{L1}}\frac{z_{L1}}{\psi_S}; \quad \varepsilon_{L0} \equiv -\frac{d\psi_S}{dz_{L0}}\frac{z_{L0}}{\psi_S}; \quad \epsilon_{L1} \equiv -\frac{d[z_{L1}m^L(z_{L1})]}{m^L(z_{L1})dz_{L1}}; \quad \epsilon_{L0} \equiv \frac{d[z_{L0}m^L(z_{L0})]}{m^L(z_{L0})dz_{L0}}.$$

We find that when the migrants' entrepreneurial rents rise following migration (i.e., the inequality (18) holds), it is always the case that $\Lambda(\Omega) < \Lambda^L(\Omega)$, as shown in Figure 6.[28] The negative competition effect in the Northern market dominates so that the innovators' rents $\pi_N(z)$ fall and the cutoff z^P rises, for any Ω. Proposition 5 follows.

Proposition 5 (Migration and innovation). *Suppose E_S/E_N is low enough that the inequality (18) holds. Then, allowing high-ability migration into the North increases z^P (the innovation set contracts).*

Proof: See Appendix.

[27] ϵ_{L0} is the elasticity of $\pi_S^L(z_{L0})$ with respect to z_{L0} and ϵ_{L1} is the elasticity of $\pi_S^L(z_{L1})$ with respect to z_{L1}.
[28] This condition is true regardless of whether the L imitation range expands or contracts in response to M.

5 IPRs versus Migration

In this section, we compare the policies' impacts on the entrepreneurial activity, income, welfare, and the composition of innovated and imitated varieties. For simplicity, we assume that $a_L > a_H/2$, which ensures that $2z_{H0}m^H(z_{H0}) - a_H m^H(a_H) > 0$ in (21) and $d\Lambda(\Omega)/d\Omega < 0$ as in Figure 6a. We find that at the aggregate level, the two policies have similar effects. Proposition 6 summarizes the results.

Proposition 6. *Both strengthening of IPRs in the South and allowing high-ability migration into the North (i) increase the quality-adjusted number of innovated varieties, ψ_N, and decrease the quality-adjusted expected number of imitated varieties, ψ_S; and (ii) increase the North's total income and welfare and decrease the South's total income and welfare.*

Proof: See Appendix.

The quality-adjusted number of innovated varieties, ψ_N given in (20), rises in response to stronger IPRs in the South because the innovation set expands into lower-quality varieties (z^P falls). With high-ability migration into the North, ψ_N also rises, but for a different reason: the mass of entrepreneurs rises in the North (in proportion to a_H), and this effect is strong enough so that ψ_N rises despite a contraction in the innovation set (z^P rises). In the South, the quality-adjusted expected number of imitated products, ψ_S, falls under both policies. With stronger IPRs, ψ_S falls because the L imitation set contracts (z_{L0} rises and z_{L1} falls). High-ability migration has more complex impacts: the mass of a_H varieties available for imitation rises but the H imitation set contracts, and the L imitation set also adjusts. We find that ψ_S falls overall when $a_L > a_H/2$, in which case the H imitation set contraction has a relatively strong effect.[29]

Assuming the migrants' earnings count towards the Northern income, we find that the Northern and Southern income levels are given by:

$$Y_N = wL_N^P + \int_{\Phi^P \setminus \Phi^M} \pi_N(z)dF(z) + \int_{\Phi_M} \mu^j(z)\pi_N^j(z)dF(z) + \pi_N^H(a_H)M; \tag{26}$$

$$Y_S = L_S^P + \sum_j \int_{z_{j0}}^{z_{j1}} \pi_S^j(z)dF(z) + \pi_S^H(a_H)M; \tag{27}$$

where $L_N^P = L_N - \int_{z^P}^1 dF(z)$ and $L_S^P = L_S(1 - g_S^H) - \int_{z_{L0}}^{z_{L1}} dF(z)$ is the mass of production workers in

[29] ψ_S rises when the L imitation set expands, but this effect is of secondary importance since migration affects L imitation only indirectly, by limiting the H imitation activity. When $a_L > a_H/2$ does not hold and $2z_{H0}m^H(z_{H0}) - a_H m^H(a_H) < 0$ in (21), ψ_S may rise.

the North and the South respectively.[30] The expressions (26)-(27) further simplify to the following:

$$Y_N = w\left(L_N - \int_{z^P}^{1} dF(z)\right) + \frac{E_N}{\sigma} + \frac{\psi_N - \psi_S}{\Psi}\frac{E_S}{\sigma}; \tag{28}$$

$$Y_S = L_S(1 - g_S^H) - \int_{z_{L0}}^{z_{L1}} dF(z) + \frac{w^{\sigma-1}\psi_S}{\Psi}\frac{E_S}{\sigma}; \tag{29}$$

where ψ_N and ψ_S are given in (20).

From Proposition 6, both policies increase income in the North and decrease income in the South, but their impact on the distribution of income within each region is critically different. National income consists of the income of production workers and entrepreneurs. With stronger IPRs, the total production income falls in the North (as the innovation set expands) but rises in the South (as the L imitation set contracts). With migration, the total production income rises in the North (because the innovation set contracts) and falls in the "less-developed" South, where IPRs protection is weak and the wage rate is low (as the L imitation set expands).

Also, both policies increase the welfare in the North and decrease the welfare in the South. This follows from the indirect utility functions, given by $V_N = Y_N - E_N + E_N/P_N$ and $V_S = Y_S - E_S + E_S/P_S$, where $P_N = p_N \psi_N^{1/(1-\sigma)}$ and $P_S = p_N[\psi_N + (w^{\sigma-1} - 1)\psi_S]^{1/(1-\sigma)}$. The welfare rises in the North because the Northern income Y_N rises and the price-quality index P_N falls, and it falls in the South because the Southern income Y_S falls and the price-quality index P_S rises.

Overall, both policies serve to promote innovation and limit imitation activity, but they differ in the margins of their impact. On the extensive margin, stronger IPRs promote innovation and limit imitation of low-quality varieties; whereas high-ability migration discourages innovation, limits high-quality imitation (due to "brain drain"), and shifts the composition of imitation towards low-quality varieties in the "less-developed" South.

It is worthwhile to more thoroughly examine the impacts on the composition of innovated and imitated varieties. Let $\tilde{\psi}_N \equiv \int_{z^P}^{1} dF(z) + M$ denote the (unadjusted) number of innovated varieties, and $\tilde{\psi}_S^L \equiv \int_{z_{L0}}^{z_{L1}} \mu^L(z) dF(z)$ and $\tilde{\psi}_S^H \equiv \int_{z_{H0}}^{z_{H1}} \mu^H(z) dF(z) + \mu^H(a_H)M$ denote the (unadjusted) number of the L and H imitated varieties, respectively. Then $\psi_N^q \equiv \psi_N/\tilde{\psi}_N$ measures the average quality of innovated varieties, and $\psi_S^{Lq} \equiv \psi_S^L/\tilde{\psi}_S^L$ and $\psi_S^{Hq} \equiv \psi_S^H/\tilde{\psi}_S^H$ measures the expected average quality of the L and H imitated varieties. To simplify the analysis, assume that Northern individuals' ability a is Pareto distributed over the range $[a_0, 1]$, with the tail index k, i.e., $G(a) = [1-(a_0/a)^k]/[1-(a_0)^k]$. Then, $F(z) = [1-(z_0/z)^k]/[1-(z_0)^k]$. Further assume that $\alpha_{H2}/\beta_{H2} > 1$ or $m^H(1) > 0$, which implies that $a_H > 1/2$ (from Lemma 1). Proposition 7 highlights the differential effects of the two policies.

[30] The migrants' earnings are given by $\pi_N^H(a_H)M = a_H M \psi_N\{E_N + [1 - \mu^H(a_H)]\xi E_S\}/\sigma$, where $d\xi/dM > 0$.

Proposition 7. *Suppose the tail index k is high. The two policies differ in their effect on the following: (i) the average quality of innovated varieties: ψ_N^q falls with stronger IPRs and rises with migration; (ii) the expected average quality of the H imitated varieties: ψ_S^{Hq} does not change with stronger IPRs and rises with migration; and (iii) the expected average quality of the L imitated varieties in the "less-developed" South: ψ_S^{Lq} rises with stronger IPRs and falls with migration.*

Proof: See Appendix.

A sufficient condition in Proposition 7 is $k \geq 4$. The two policies differ in their effect on the composition of innovation. With stronger IPRs protection in the South, the innovation set expands into the low-quality varieties. Both the quality-adjusted and the unadjusted number of innovated varieties rise in response, but the unadjusted number $\tilde{\psi}_N$ rises relatively more; thus, the average quality ψ_N^q falls. With high-ability migration into the North, the average quality of innovated varieties rises for two reasons: the innovation set contracts away from low-quality varieties, and the number of high-quality (a_H) innovated varieties rises.

The composition of imitation is also impacted differently. Stronger IPRs in the South increase the expected average quality of the L imitated varieties (as the L imitation set contracts) but do not affect the expected average quality of the H imitated varieties (since ψ_S^H and $\tilde{\psi}_S^H$ fall in the same proportion). High-ability migration, by contrast, limits imitation of high-quality varieties (as the H imitation set contracts) but also promotes high-quality imitation as the mass of a_H imitated varieties rises; the expected average quality of the H imitated varieties rises in response.

6 Conclusion

This paper introduced an occupational choice model of innovative North and imitative South with two dimensions of heterogeneity: product quality and entrepreneurial ability. This framework allowed us to study how two policies (strengthening of IPRs in the South and opening the North to high-ability migration as means of preempting imitation) impact innovation activity in the North and imitation activity in the South. The policies' impacts depend on the endogenous entrepreneurship decisions and the intensity of competition between innovated and imitated varieties.

The model predicts that the two policies have critically different implications for the entrepreneurial activity and income distribution in each region. Opening the North to high-ability migration directly limits the imitation of high-quality products; but in the "less-developed" South, where IPRs protection is weak and wage rate is low, it also promotes the imitation of low-quality products. A strengthening of IPRs in the South, by contrast, limits low-quality imitation and does not affect the set of high-quality imitated products. Furthermore with high-ability migration from the "less-developed" South, the aggregate production income falls in the South and rises in the

North, whereas a strengthening of IPRs in the South has the opposite effect.

The findings suggest that the North's high-ability immigration policy could be an attractive alternative to the policy of imposing stronger IPRs in developing economies when the goal is to combat imitation and promote innovation. Improved migration prospects for high-ability entrepreneurs is not a zero-sum game: the rents of low-ability entrepreneurs could also rise in the South, as well as the average quality of products in the high-ability imitation sector.

In further research, the model could be extended to allow for innovation in the South. Such model would be better suited to study the policies' impacts on an emerging economy (e.g., China) which possesses a critical level of complementary research, technological, and marketing assets to enable the local entrepreneurs to absorb foreign technology. The strength of IPRs in this "more advanced" Southern economy would still be below the global optimal level, and the North would have an incentive to push for global IPRs reforms (Lai and Qiu, 2003). The reforms of IPRs could facilitate the South's transition from an imitative to innovative economy, and this transition could diminish the incentive of high-ability entrepreneurs to migrate to the North. This interplay between the two policies would be interesting to explore, but the role of migration policy in limiting imitation—which is the focus of this paper—would become less pronounced. Finally, our model's predictions could be tested using data on inventor migration flows in Miguelez and Finks (2013) and the IAB Brain Drain data in Brüker et al. (2013).

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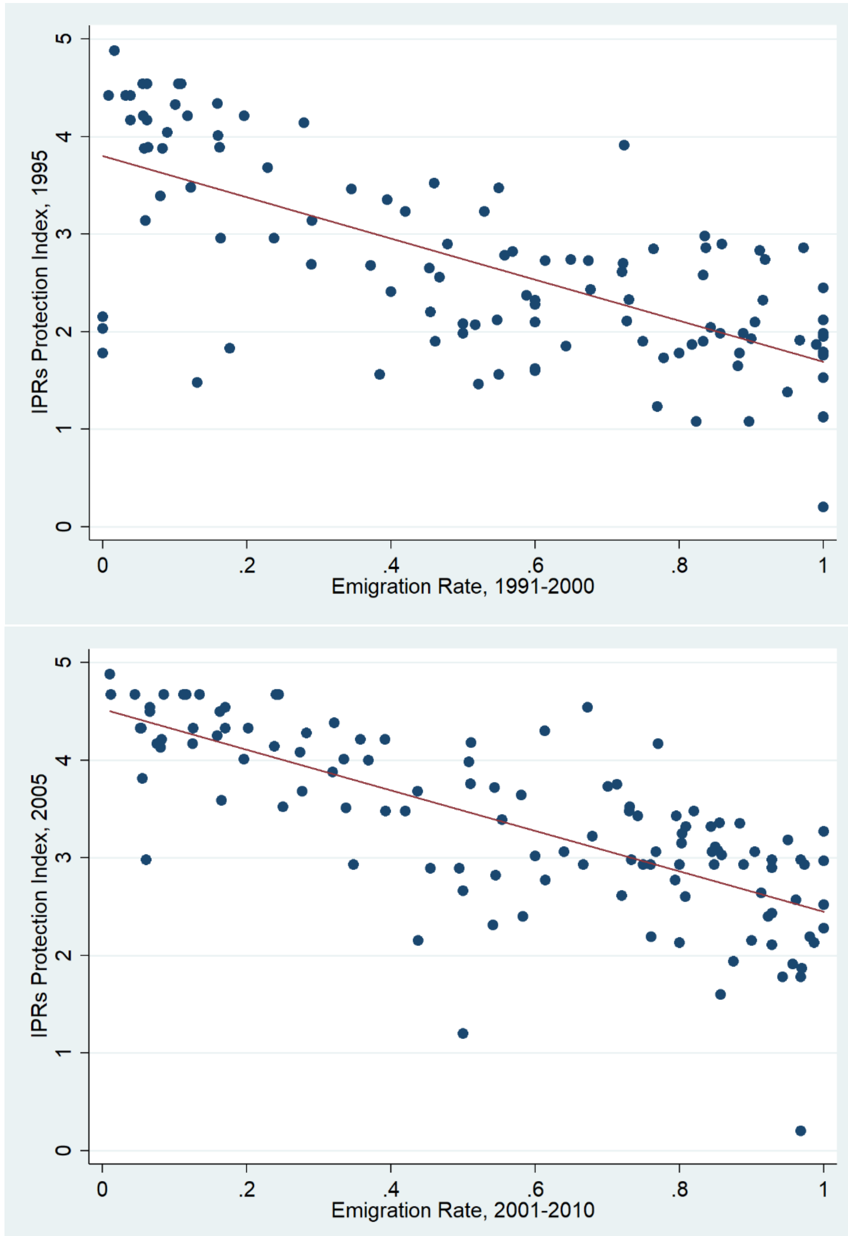

Figure 1: **IPRs protection and inventor emigration rate**

Notes: The inventor emigration rate of origin country i is defined as $diaspora_i/(diaspora_i + residents_i)$, where $diaspora_i$ is the number of national inventors of country i residing abroad and $residents_i$ is the number of inventors residing in country i (including national of country i and immigrants). These data are from Miguelez and Finks (2013). The index of IPRs protection measures the stringency of patent rights, based on five measures of patent laws (coverage, membership in international patent treaties, provisions against losses of protection, enforcement mechanisms, and duration of protection). These data are from Park (2008).

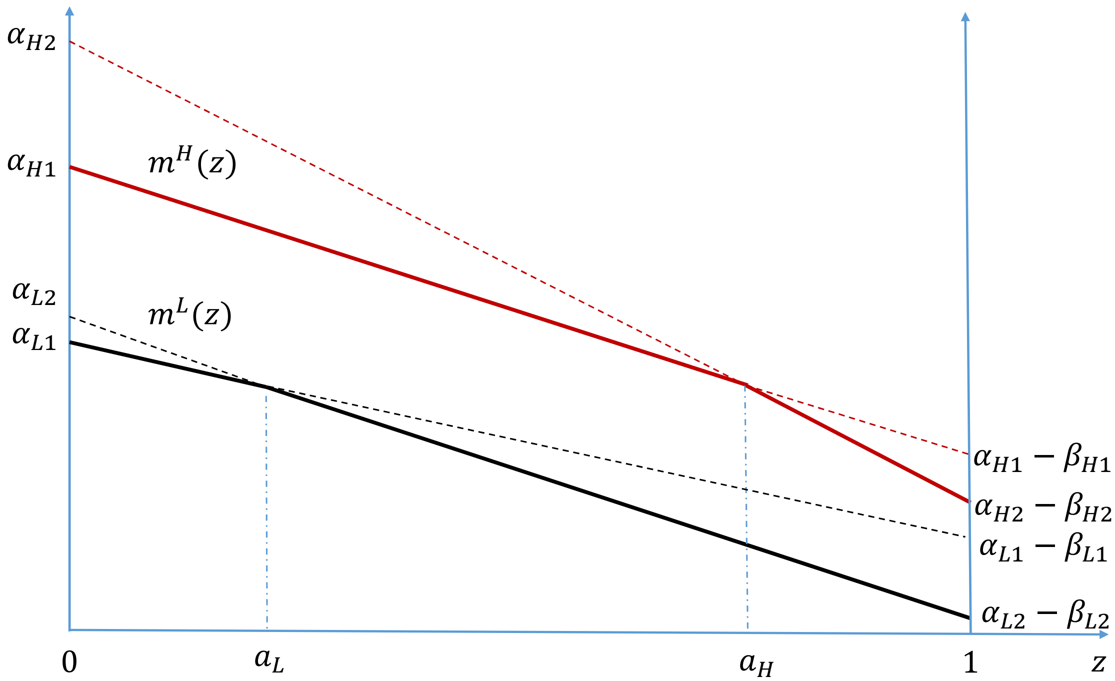

Figure 2: **Imitation rates**

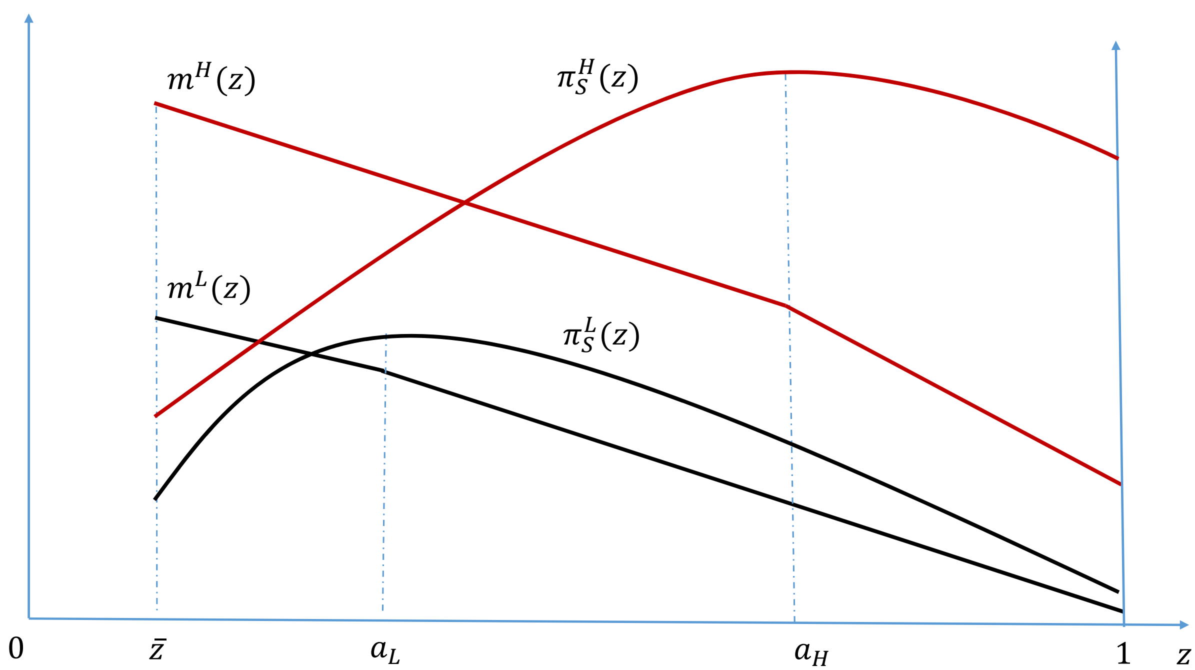

Figure 3: **Imitators' rents**

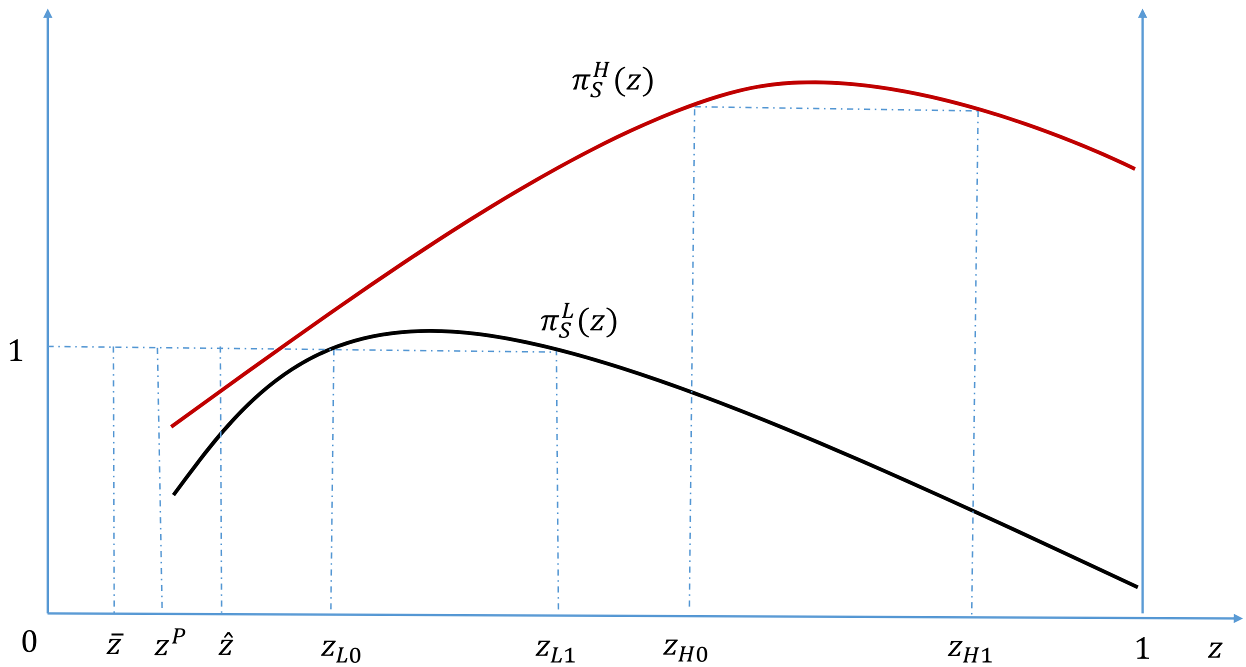

Figure 4: **Equilibrium with no overlap in imitation**

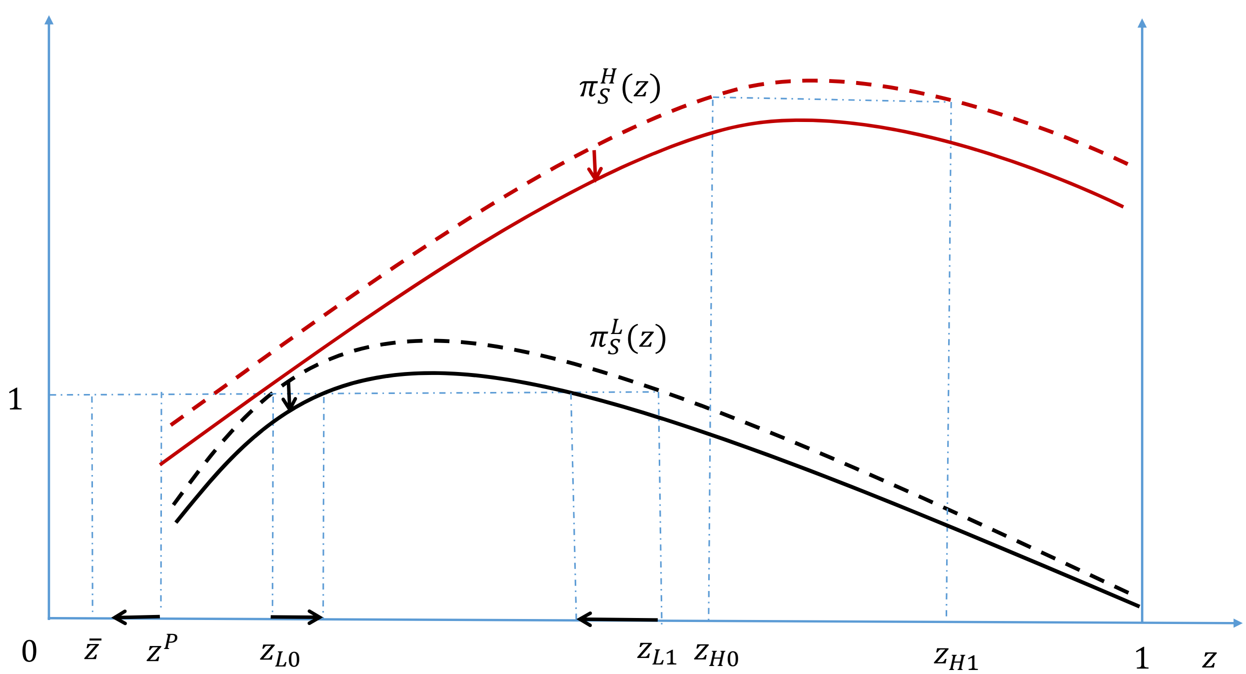

Figure 5: **Strengthening IPRs in the South**

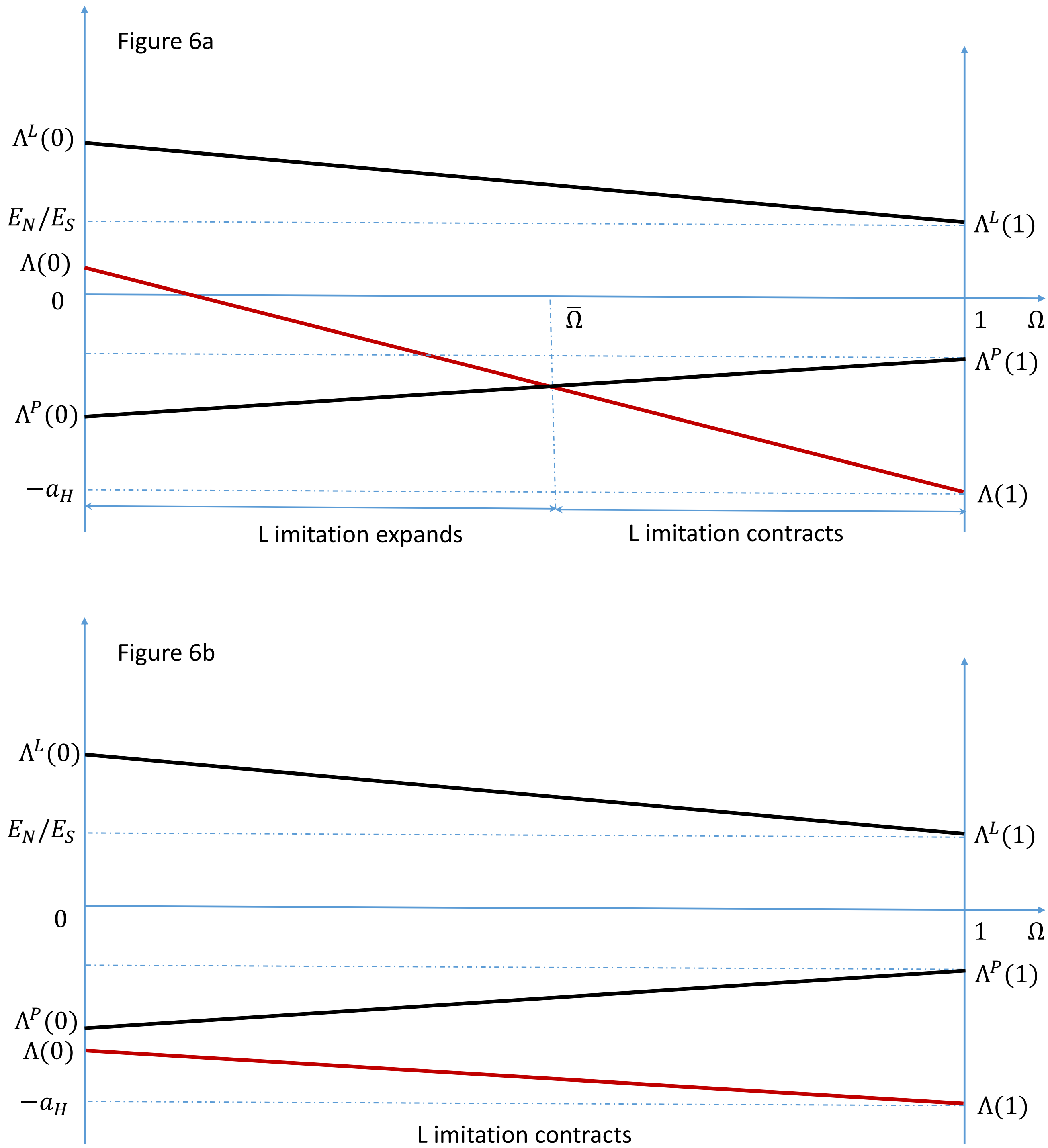

Figure 6: **Migration and imitation**

Appendix

Proof of Proposition 1

From $G(z) \equiv zE_N - \sigma w \int_z^1 x dF(x)$, we have $G(0) < 0$, $G(1) > 0$, and $dG(z)/dz > 0$. Thus, there exists a unique equilibrium with $0 < \hat{z} < 1$ such that $G(\hat{z}) = 0$. From $G^F(z) \equiv z(E_N + E_S) - \sigma w \int_z^1 x dF(x)$ and above, we obtain $G^F(0) < 0$, $G^F(\hat{z}) > 0$, and $dG^F(z)/dz > 0$. Thus, there exists a unique equilibrium with $0 < \bar{z} < \hat{z}$ such that $G^F(\bar{z}) = 0$.

Proof of Lemma 2

$\hat{z} < z_{L0}$ iff $\pi_S^L(\hat{z}) < \pi_S^L(z_{L0})$. Using $\pi_S^L(z) = z\mu^L(z)(p_S/P_S)^{1-\sigma}E_S/\sigma$, where $\mu^j(z) = (1-\Omega)(\alpha_{L1} - \beta_{L1}z)$ for $z \leq a_j$, we find that $\pi_S^L(\hat{z}) < \pi_S^L(z_{L0})$ requires $\hat{z} + z_{L0} < a_{L1}/\beta_{L1}$, which holds if $\alpha_{L1}/\beta_{L1} > 1$ because $\hat{z} < 1/2$ from Section 2.1 and $z_{L0} \leq a_L \leq \alpha_{L1}/(2\beta_{L1})$ from Lemma 1.

Proof of Lemma 3

The cutoffs z_{j0} and z_{j1} for $j \in \{L, H\}$ are defined by the following four conditions:

$$\int_{z_{H0}}^{z_{H1}} dF(z) - g_S^H L_S = 0, \tag{A1}$$

$$(\alpha_{H1} - \beta_{H1}z_{H0})z_{H0} - (\alpha_{H2} - \beta_{H2}z_{H1})z_{H1} = 0, \tag{A2}$$

$$(\alpha_{L1} - \beta_{L1}z_{L0})z_{L0} - (\alpha_{L2} - \beta_{L2}z_{L1})z_{L1} = 0, \tag{A3}$$

$$z_{L0}\mu^L(z_{L0})\frac{w^{\sigma-1}E_S}{\psi_N + (w^{\sigma-1}-1)\psi_S} - \sigma = 0, \tag{A4}$$

where (A2)-(A3) follow from $\pi_S^j(z_{j0}) = \pi_S^j(z_{j1})$ and (A4) follows from $\pi_S^L(z_{L0}) = 1$.

We need to find the sufficient condition for $z_{L1} < z_{H0}$. Since $z_{H1} < 1$ from Lemma 2, it follows from (A2) that $z_{H0} > \tilde{z}_{H0}$, where $\tilde{z}_{H0}$ solves $(\alpha_{H1} - \beta_{H1}\tilde{z}_{H0})\tilde{z}_{H0} - (\alpha_{H2} - \beta_{H2}) = 0$. Also since $z_{L0} > \hat{z}$, it follows from (A3) that $z_{L1} < \tilde{z}_{L1}$, where $\tilde{z}_{L1}$ solves $(\alpha_{L2} - \beta_{L2}\tilde{z}_{L1})\tilde{z}_{L1} - (\alpha_{L1} - \beta_{L1}\hat{z})\hat{z} = 0$. Thus, $z_{L1} < z_{H0}$ if $\tilde{z}_{L1} < \tilde{z}_{H0}$, which requires a sufficiently high $a_H = (\alpha_{H2} - \alpha_{H1})/(\beta_{H2} - \beta_{H1})$ and low $a_L = (\alpha_{L2} - \alpha_{L1})/(\beta_{L2} - \beta_{L1})$, for a given $E_N/(\sigma w)$.

The Overlapping Case

Lemma A.1 establishes the sufficient conditions for the overlapping case.

Lemma A.1. *There exists $\tilde{g}_S^H$ such that in equilibrium, the L entrepreneurs imitate varieties $z \in [z_{L0}, z_{L1}]$, the H entrepreneurs imitate varieties $z \in [z_{H0}, z_{H1}]$, and the two imitation sets* *overlap, $z_{L1} > z_{H0}$, if $g^H_S \geq \tilde{g}^H_S$, a_L is high and Ω is low.*

Proof. These two conditions are sufficient for the overlapping case: (i) $g^H_S \geq \tilde{g}^H_S$, where $\tilde{g}^H_S \equiv \int_{a_L}^1 dF(z)/L_S$ and (ii) $[1-\mu^H(a_L)]\pi^L_S(a_L) > 1$. Under these conditions, the H imitation set, given by $[z_{H0}, 1]$ with $z_{H0} < a_L$, overlaps with the L imitation set, given by $[z_{L0}, z_{L1}]$ with $a_L \in (z_{L0}, z_{L1})$. Substituting for $\pi^L_S(a_L)$, we rewrite $[1-\mu^H(a_L)]\pi^L_S(a_L) > 1$ as follows: $[1-\mu^H(a_L)]a_L\mu^L(a_L)w^{\sigma-1}E_S > \sigma[\psi_N + (w^{\sigma-1}-1)\psi_S]$. Since $\mu^H(a_L) < (1-\Omega)\alpha_{H1}$ and $\psi_N + (w^{\sigma-1}-1)\psi_S < w^{\sigma-1}\psi_N$, this inequality holds if

$$[1-(1-\Omega)\alpha_{H1}]a_L(1-\Omega)m^L(a_L)w^{\sigma-1}E_S > \sigma\psi_N, \tag{A5}$$

where $\psi_N = \bar{z}(E_N + E_S)/(\sigma w)$ from $G^F(\bar{z}) = 0$. If $\Omega < 1 - 1/(2\alpha_{H1})$, the left hand side in (A5) rises as Ω rises and so, (A5) holds if it holds for $\Omega = 0$, i.e., if $(1-\alpha_{H1})a_L m^L(a_L)$ is high. Thus, $[1-\mu^H(a_L)]\pi^L_S(a_L) > 1$ requires a high a_L and a low Ω, for a given α_{H1}, E_N/E_S and w.[31] ■

Define z'_{j0} and z'_{j1} as in the non-overlapping case and use $\pi^j_S(z)$ to denote j's rents in the entire range $z > \bar{z}$. We have $\pi^H_S(z'_{H0}) = \pi^H_S(z'_{H1})$ and $\pi^L_S(z'_{L0}) = \pi^L_S(z'_{L1}) = 1$. If the two imitation sets overlap, then $z'_{H0} < z'_{L1}$ and L and H entrepreneurs face competition in imitating variety $z \in [z'_{H0}, z'_{L1}]$. This competition reduces the imitators' expected rents below $\pi^j_S(z)$. Suppose, for example, that for any $z \in [z'_{H0}, z'_{L1}]$, one H and one L entrepreneur try to imitate the same variety. If only one entrepreneur succeeds, that imitator will compete with the innovator and will receive the *ex-post* expected rents $\pi^j_S(z)$. If both entrepreneurs succeed, they will also compete in Bertrand between themselves, driving their *ex-post* rents to zero (since their marginal cost is the same). When the likelihood of imitation is $\mu^j(z)$, the expected rents in the range $[z'_{H0}, z'_{L1}]$ are equal to $[1-\mu^L(z)]\pi^H_S(z)$ for the H imitator and $[1-\mu^H(z)]\pi^L_S(z)$ for the L imitator. From the H imitator's perspective, the expected rents rise with quality in the range of low-quality varieties, because $\mu^L(z)$ falls while $\pi^H_S(z)$ rises as z rises. From the L imitator's perspective, by contrast, the expected rents fall with product quality in the range of high-quality varieties if $\alpha_{L1}/(\beta_{L1} + 0.5\beta_{L2}) < a_L$.[32] This sufficient condition requires a low α_{L2}/β_{L2} to ensure that as z rises, $\pi^L_S(z)$ falls faster than $\mu^H(z)$ does. From Lemma A.1 and its proof, we obtain Proposition A.1.

Proposition A.1. *In the overlapping case, where $\Phi^M = [z_{L0}, z_{H1}]$ and $z_{L1} > z_{H0}$, the end points defining the two intervals, z_{j0} and z_{j1}, are determined by (11), $\pi^L_S(z_{L0}) = [1-\mu^H(z_{L1})]\pi^L_S(z_{L1}) = 1$ and $[1-\mu^L(z_{H0})]\pi^H_S(z_{H0}) = \pi^H_S(z_{H1}) > 1$.*

[31] Note that $d[a_L m^L(a_L)]/da_L > 0$ because $a_L < \alpha_{L1}/(2\beta_{L1})$.

[32] $[1-\mu^H(z)]\pi^L_S(z)$ falls as z rises in the range $(a_L, a_H]$ iff $\beta_{H1}z(\alpha_{L2} - \beta_{L2}z) < [1/(1-\Omega) - \alpha_{H1} + \beta_{H1}z](2\beta_{L2}z - \alpha_{L2})$, for which the condition $2\alpha_{L2}/(3\beta_{L2}) < a_L$ is sufficient since $\Omega < 1$, $\alpha_{H1} < 1$, and $z > a_L$. Using $\alpha_{L2} = \alpha_{L1} + (\beta_{L2} - \beta_{L1})a_L$, we rewrite $2\alpha_{L2}/(3\beta_{L2}) < a_L$ as follows: $\alpha_{L1}/(\beta_{L1} + 0.5\beta_{L2}) < a_L$.

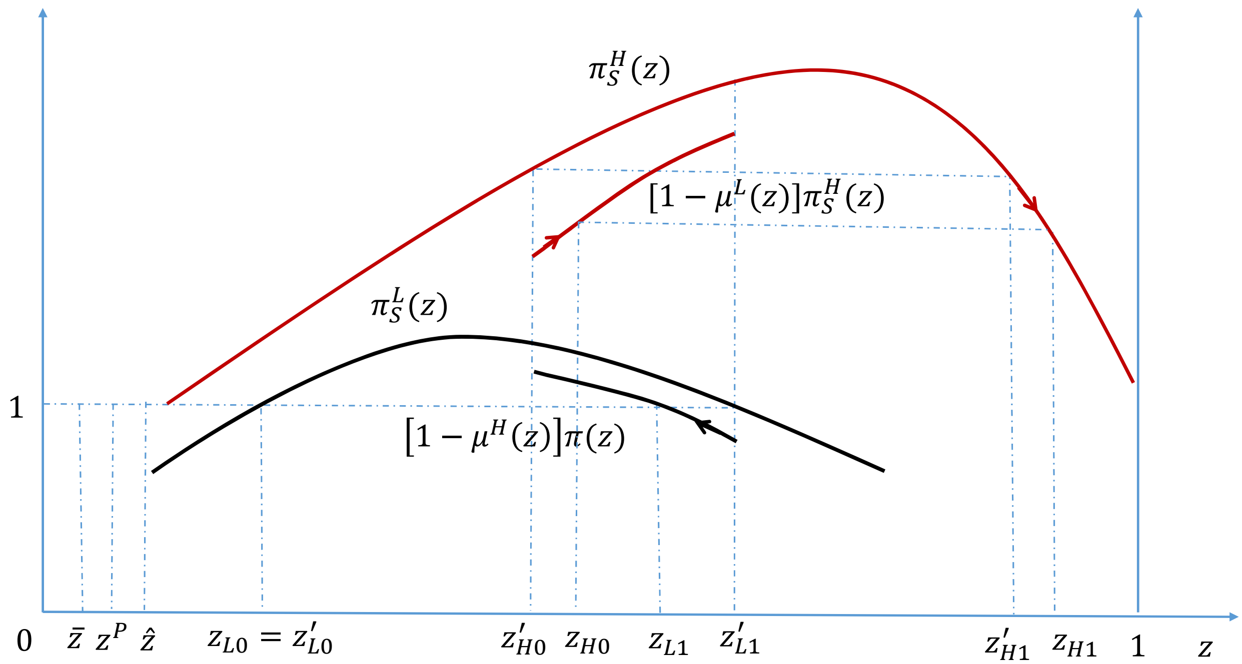

Figure A1: **Equilibrium with overlap in imitation**

Figure A.1 shows one overlapping equilibrium. The following "thought experiment" provides intuition to this equilibrium allocation. Suppose the L and H entrepreneurs enter the market in a sequential manner. In equilibrium, the H and L entrepreneurs will never compete in the entire range $[z'_{H0}, z'_{L1}]$. Consider an H entrepreneur outside the no-competition range $[z'_{L1}, z'_{H1}]$. This entrepreneur can either imitate a variety to the immediate right of z'_{H1}, say $z_h^+(> z'_{H1})$, or a variety within $[z'_{H0}, z'_{L1}]$, say z_h^-. In the latter case, it will choose the highest possible quality, i.e., z_h^- will be to the immediate left of z'_{L1}, because $[1-\mu^L(z)]\pi_S^H(z)$ is an increasing function of z. The H entrepreneurs will give up some varieties in $[z'_{H0}, z'_{L1}]$ to ensure that the mass of H imitators is equal to the mass innovators over the H imitation range. Due to the monotonicity of rents within this range, the H entrepreneurs will give up varieties to the immediate right of z'_{H0}. The cutoffs z_{H0} and z_{H1} are determined by (11) and $[1-\mu^L(z_{H0})]\pi_S^H(z_{H0}) = \pi_S^H(z_{H1}) > 1$. Next, consider an L entrepreneur outside the non-competition range $[z'_{L0}, z'_{H0}]$. This entrepreneur can imitate a variety within $[z'_{H0}, z'_{L1}]$, say z_l^+. It will choose the lowest possible quality, i.e., z_l^+ will be to the immediate right of z'_{H0}, because $[1-\mu^L(z)]\pi_S^L(z)$ is a decreasing function of z. Due to the monotonicity of rents within $[z'_{H0}, z'_{L1}]$, the L entrepreneurs will give up varieties to the immediate left of z'_{L1} to ensure that the L imitators' expected rents are not below the wage rate of one. The cutoffs z_{L0} and z_{L1} are determined by $\pi_S^L(z_{L0}) = [1-\mu^H(z_{L1})]\pi_S^L(z_{L1}) = 1$. The innovator $z \in [z_{H0}, z_{L1}]$ competes with both L and H imitators and so, earns rents given by

$$\pi_N^{LH}(z) = \frac{z}{\sigma}\left\{\frac{E_N}{\psi_N} + \frac{[1-\mu^L(z)][1-\mu^H(z)]E_S}{\psi_N + (w^{\sigma-1}-1)\psi_S}\right\}, \quad \text{where} \quad \pi_N^{LH\prime}(z) > 0 \quad \text{and} \quad \pi_N^{LH}(z) > w.$$

Proof of Proposition 2

Since $\hat{z} < z_{L0}$, innovator $z \in (\bar{z}, \hat{z}]$ does not face imitation and earns the rents given by

$$\pi_N(z) = \frac{z}{\sigma}\left[\frac{E_N}{\psi_N} + \frac{E_S}{\psi_N + (w^{\sigma-1}-1)\psi_S}\right], \quad \text{where} \quad \xi \equiv \frac{\psi_N}{\psi_N + (w^{\sigma-1}-1)\psi_S} < 1. \tag{A6}$$

Define $G^P(z) \equiv z(E_N + \xi E_S) - \sigma w \int_z^1 x dF(x)$. Since $dG^P(z)/dz > 0$, $G^P(\bar{z}) < 0$ and $G^P(\hat{z}) > 0$ (from Proposition 1), there must exist a unique equilibrium with $z^P \in (\bar{z}, \hat{z})$ such that $G^P(z^P) = 0$.

Innovator z in the set $\Phi^P \setminus \Phi^M = [z^P, z_{L0}) \cup (z_{L1}, z_{H0}) \cup (z_{H1}, 1]$ also earns the rents (A6). Innovator $z \in \Phi^M$ risks imitation with probability $\mu^j(z)$ and earns the expected rents given by

$$\pi_N(z) = \frac{z}{\sigma}\left\{\frac{E_N}{\psi_N} + \frac{[1-\mu^j(z)]E_S}{\psi_N + (w^{\sigma-1}-1)\psi_S}\right\}. \tag{A7}$$

It is true that $\pi_N(z) > w$ for any $z > \hat{z}$.

Labour and Income Composition

In a closed Northern economy, the mass of individuals L_N is composed of $L_N^I = \int_{\hat{z}}^1 dF(z)$ entrepreneurs and L_N^P workers, of which $L_N^{PD} = \int_{\hat{z}}^1 c_N(z)dF(z) = E_N/p_N$ produce differentiated varieties and $L_N^{PH} = (Y_N - E_N)/w$ produce the homogeneous good.[33] The total income is given by $Y_N = wL_N^P + \int_{\hat{z}}^1 \pi(z)dF(z) = w[L_N - \int_{\hat{z}}^1 dF(z)] + E_N/\sigma$.

Consider now an open trading economy. In the South, L_S is composed of $f_S^L + g_S^H L_S$ imitators and L_S^P workers, of which L_S^{PD} produce imitated varieties, and L_S^{PH} and L_X^{PH} produce the homogeneous good for domestic consumption and exports, respectively. We have $L_S^{PH} = Y_S - E_S$ and

$$L_S^{PD} = \sum_j \int_{z_{j0}}^{z_{j1}} \mu^j(z)c_S(z)dF(z) = \left[\frac{\psi_S w^{\sigma-1}}{\psi_N + \psi_S(w^{\sigma-1}-1)}\right]\frac{E_S}{p_S}.$$

Balanced trade requires the value of Southern homogeneous good exports to be equal to the value

[33] The homogeneous good supply is equal to $c_N^s(v_0) = wL_N^{PH}$ and its demand is equal to $c_N^d(v_0) = Y_N - \int_{\hat{z}}^1 p_N c_N(z)dF(z) = Y_N - E_N$. In equilibrium, $L_N^{PH} = (Y_N - E_N)/w$.

of its innovated goods imports: $L_X^{PH} = \int_{z^P}^1 p_X c_X(z)dF(z) - \int_{\Phi_M} \mu^j(z) p_X c_X(z) dF(z)$ or

$$L_X^{PH} = \left[\frac{\psi_N - \psi_S}{\psi_N + \psi_S(w^{\sigma-1} - 1)}\right] E_S. \tag{A8}$$

The total income in the South is given by $Y_S = L_S^P + \sum_j \int_{z_{j0}}^{z_{j1}} \pi_S^j(z)dF(z)$ or

$$Y_S = L_S(1 - g_S^H) - \int_{z_{L0}}^{z_{L1}} dF(z) + \left[\frac{\psi_S w^{\sigma-1}}{\psi_N + \psi_S(w^{\sigma-1} - 1)}\right] \frac{E_S}{\sigma}. \tag{A9}$$

In the North, L_N is composed of $\int_{z^P}^1 dF(z)$ innovators and L_N^P workers, of which L_N^{PH} produce the homogeneous good, and L_N^{PD} and L_X^{PD} produce innovated varieties for domestic consumption and exports. We have $L_N^{PH} = (Y_N - E_N - L_X^{PH})/w$, $L_N^{PD} = \int_{z^P}^1 c_N(z)dF(z) = E_N/p_N$, and

$$L_X^{PD} = \int_{z^P}^1 c_X(z)dF(z) - \sum_j \int_{z_{j0}}^{z_{j1}} \mu^j(z) c_X(z) dF(z) = \left[\frac{\psi_N - \psi_S}{\psi_N + (w^{\sigma-1} - 1)\psi_S}\right] \frac{E_S}{p_N}.$$

The Northern income is $Y_N = wL_N^P + \int_{\Phi^P \setminus \Phi^M} \pi_N(z)dF(z) + \int_{\Phi_M} \mu^j(z)\pi_N^j(z)dF(z)$ or

$$Y_N = w\left(L_N - \int_{z^P}^1 dF(z)\right) + \frac{E_N}{\sigma} + \left[\frac{\psi_N - \psi_S}{\psi_N + (w^{\sigma-1} - 1)\psi_S}\right] \frac{E_S}{\sigma}. \tag{A10}$$

Proof of Proposition 3

The equilibrium is defined by (A1)-(A4) and the following condition:

$$G^P \equiv z^P\left(\frac{E_N}{\psi_N} + \frac{E_S}{\Psi}\right) - \sigma w = 0, \quad \text{where} \quad \Psi \equiv \psi_N + (w^{\sigma-1} - 1)\psi_S \tag{A11}$$

It follows from (A1)-(A2) that $dz_{H0}/d\Omega = dz_{H1}/d\Omega = 0$. Next, we show that $dz^P/d\Omega < 0$, $dz_{L0}/d\Omega > 0$ and $dz_{L1}/d\Omega < 0$. Totally differentiating (A3), (A4), and (A11), we obtain:

$$\begin{bmatrix} F_P^L & F_{L0}^L & F_{L1}^L \\ 0 & G_{L0}^L & G_{L1}^L \\ G_P^P & G_{L0}^P & G_{L1}^P \end{bmatrix} \begin{bmatrix} dz^P/d\Omega \\ dz_{L0}/d\Omega \\ dz_{L1}/d\Omega \end{bmatrix} = \begin{bmatrix} -F_\Omega^L \\ 0 \\ -G_\Omega^P \end{bmatrix}, \quad \text{where}$$

$$F_\Omega^L = -z_{L0}\mu^L(z_{L0}) \frac{w^{\sigma-1}E_S}{\Psi^2} \frac{\psi_N}{1 - \Omega} < 0, \tag{A12}$$

$$F_P^L = z_{L0}\mu^L(z_{L0}) \frac{w^{\sigma-1}E_S}{\Psi^2} \left(-\frac{d\psi_N}{dz^P}\right) > 0, \tag{A13}$$

$$F^L_{L0} = \frac{d[z_{L0}\mu^L(z_{L0})]}{dz_{L0}}\frac{w^{\sigma-1}E_S}{\Psi} + z_{L0}\mu^L(z_{L0})(w^{\sigma-1}-1)\frac{w^{\sigma-1}E_S}{\Psi^2}\left(-\frac{d\psi_S}{dz_{L0}}\right) > 0, \tag{A14}$$

$$F^L_{L1} = -z_{L0}\mu^L(z_{L0})(w^{\sigma-1}-1)\frac{w^{\sigma-1}E_S}{\Psi^2}\left(\frac{d\psi_S}{dz_{L1}}\right) < 0, \tag{A15}$$

$$G^P_\Omega = z^P(w^{\sigma-1}-1)\frac{E_S}{\Psi^2}\frac{\psi_S}{1-\Omega} > 0, \tag{A16}$$

$$G^P_P = \frac{E_N}{\psi_N} + \frac{E_S}{\Psi} + z^P\left(\frac{E_N}{\psi_N^2} + \frac{E_S}{\Psi^2}\right)\left(-\frac{d\psi_N}{dz^P}\right) > 0, \tag{A17}$$

$$G^P_{L0} = z^P(w^{\sigma-1}-1)\frac{E_S}{\Psi^2}\left(-\frac{d\psi_S}{dz_{L0}}\right) > 0, \tag{A18}$$

$$G^P_{L1} = -z^P(w^{\sigma-1}-1)\frac{E_S}{\Psi^2}\left(\frac{d\psi_S}{dz_{L1}}\right) < 0, \tag{A19}$$

$$G^L_{L0} = \alpha_{L1} - 2\beta_{L1}z_{L0} > 0, \tag{A20}$$

$$G^L_{L0} = 2\beta_{L2}z_{L1} - \alpha_{L2} > 0. \tag{A21}$$

We find that $D \equiv G^L_{L1}(F^L_{L0}G^P_P - F^L_P G^P_{L0}) + G^L_{L0}(F^L_P G^P_{L1} - F^L_{L1}G^P_P) > 0$, where the terms in the brackets are positive from (A13)-(A19) and $G^L_{L1} > 0$ and $G^L_{L0} > 0$ from (A20)-(A21). We have

$$\frac{dz^P}{d\Omega} = \frac{1}{D}\begin{bmatrix} -F^L_\Omega & F^L_{L0} & F^L_{L1} \\ 0 & G^L_{L0} & G^L_{L1} \\ -G^P_\Omega & G^P_{L0} & G^P_{L1} \end{bmatrix}, \quad \frac{dz_{L0}}{d\Omega} = \frac{1}{D}\begin{bmatrix} F^L_P & -F^L_\Omega & F^L_{L1} \\ 0 & 0 & G^L_{L1} \\ G^P_P & -G^P_\Omega & G^P_{L1} \end{bmatrix}, \quad \frac{dz_{L1}}{d\Omega} = \frac{1}{D}\begin{bmatrix} F^L_P & F^L_{L0} & -F^L_\Omega \\ 0 & G^L_{L0} & 0 \\ G^P_P & G^P_{L0} & -G^P_\Omega \end{bmatrix}.$$

It follows that $dz^P/d\Omega = [F^L_\Omega(G^L_{L1}G^P_{L0} - G^L_{L0}G^P_{L1}) - G^P_\Omega(F^L_{L0}G^L_{L1} - F^L_{L1}G^L_{L0})]/D < 0$; $dz_{L0}/d\Omega = G^L_{L1}(F^L_P G^P_\Omega - F^L_\Omega G^P_P)/D > 0$; and $dz_{L1}/d\Omega = -G^L_{L0}(F^L_P G^P_\Omega - F^L_\Omega G^P_P)/D < 0$, where $F^L_\Omega < 0$, $G^P_\Omega > 0$, $G^L_{L1} > 0$, $G^L_{L0} > 0$ and all terms in the round brackets are positive.

Proof of Proposition 4

When $M > 0$, the equilibrium is defined by the following conditions:

$$F^H \equiv \int_{z_{H0}}^{z_{H1}} dF(z) + 2M - g^H_S L_S = 0, \tag{A22}$$

$$G^H \equiv (\alpha_{H1} - \beta_{H1}z_{H0})z_{H0} - (\alpha_{H2} - \beta_{H2}z_{H1})z_{H1} = 0, \tag{A23}$$

$$G^L \equiv (\alpha_{L1} - \beta_{L1}z_{L0})z_{L0} - (\alpha_{L2} - \beta_{L2}z_{L1})z_{L1} = 0, \tag{A24}$$

$$F^L \equiv z_{L0}\mu^L(z_{L0})\frac{w^{\sigma-1}E_S}{\Psi} - \sigma = 0, \tag{A25}$$

$$G^P \equiv z^P \left(\frac{E_N}{\psi_N} + \frac{E_S}{\Psi} \right) - \sigma w = 0, \tag{A26}$$

where $\Psi \equiv \psi_N + (w^{\sigma-1} - 1)\psi_S$. Totally differentiating (A22)-(A23), we obtain

$$\begin{bmatrix} F^H_{H0} & F^H_{H1} \\ G^H_{H0} & G^H_{H1} \end{bmatrix} \begin{bmatrix} dz_{H0}/dM \\ dz_{H1}/dM \end{bmatrix} = \begin{bmatrix} -F^H_M \\ 0 \end{bmatrix}.$$

Hence, $dz_{H0}/dM = -F^H_M G^H_{H1}/D^H > 0$ and $dz_{H1}/dM = F^H_M G^H_{H0}/D^H < 0$, where $D^H = F^H_{H0}G^H_{H1} - F^H_{H1}G^H_{H0} < 0$, $F^H_{H0} < 0$, $F^H_{H1} > 0$, $F^H_M > 0$ from (A22), and $G^H_{H0} > 0$ and $G^H_{H1} > 0$ from (A23).

Next, totally differentiating (A25)-(A26), we obtain

$$\begin{bmatrix} F^L_P & F^L_{L0} & F^L_{L1} \\ 0 & G^L_{L0} & G^L_{L1} \\ G^P_P & G^P_{L0} & G^P_{L1} \end{bmatrix} \begin{bmatrix} dz^P/dM \\ dz_{L0}/dM \\ dz_{L1}/dM \end{bmatrix} = \begin{bmatrix} -F^L_M \\ 0 \\ -G^P_M \end{bmatrix}, \tag{A27}$$

$$\text{where} \quad F^L_M = \Lambda(\Omega) z_{L0} \mu^L(z_{L0}) \frac{w^{\sigma-1}E_S}{\Psi^2}, \quad G^P_M = z^P \left[\Lambda(\Omega)\frac{E_S}{\Psi^2} - a_H \frac{E_N}{\psi_N^2} \right], \quad \text{and} \tag{A28}$$

$$\Lambda(\Omega) = (w^{\sigma-1} - 1)[2z_{H0}\mu^H(z_{H0}) - a_H\mu^H(a_H)] - a_H, \tag{A29}$$

since $z_{H0}\mu^H(z_{H0}) = z_{H1}\mu^H(z_{H1})$.[34] Thus, $dz_{L0}/dM = G^L_{L1}(F^L_P G^P_M - F^L_M G^P_P)/D$ and $dz_{L1}/dM = -G^L_{L0}(F^L_P G^P_M - F^L_M G^P_P)/D$, where $D \equiv G^L_{L1}(F^L_{L0}G^P_P - F^L_P G^P_{L0}) + G^L_{L0}(F^L_P G^P_{L1} - F^L_{L1}G^P_P) > 0$ from the proof of Proposition 3. Since $G^L_{L0} > 0$ and $G^L_{L1} > 0$, we have $dz_{L0}/dM < 0$ and $dz_{L1}/dM > 0$ if $F^L_P G^P_M < F^L_M G^P_P$. Using (A13), (A17), (A28), and $\xi = \psi_N/\Psi$ and simplifying, we rewrite $F^L_P G^P_M < F^L_M G^P_P$ as follows:

$$\Lambda(\Omega)\left(1 + \frac{\xi E_S}{E_N}\right) + (\Lambda + a_H)\varepsilon^P > 0, \quad \text{where} \quad \varepsilon^P \equiv -\frac{d\psi_N}{dz^P}\frac{z^P}{\psi_N} = z^P f(z^P)\frac{z^P}{\psi_N}. \tag{A30}$$

Define $\Lambda^P(\Omega)$ by $\Lambda^P(\Omega)(1 + \xi E_S/E_N) + [\Lambda^P(\Omega) + a_H]\varepsilon^P = 0$. That is,

$$\Lambda^P(\Omega) = -\frac{a_H\varepsilon^P}{1 + \varepsilon^P + \xi E_S/E_N}. \tag{A31}$$

Then, $dz_{L0}/dM < 0$ and $dz_{L1}/dM > 0$ if $\Lambda(\Omega) > \Lambda^P(\Omega)$, and $dz_{L0}/dM \geq 0$ and $dz_{L1}/dM \leq 0$ if

[34] Since $z_{H0}\mu^H(z_{H0}) = z_{H1}\mu^H(z_{H1})$, we have:

$$-\frac{d\psi_S}{dz_{H0}}\frac{dz_{H0}}{dM} - \frac{d\psi_S}{dz_{H1}}\frac{dz_{H1}}{dM} = 2z_{H0}\mu^H(z_{H0}).$$

$\Lambda(\Omega) \leq \Lambda^P(\Omega)$. Assuming $F(z) = [1-(z_0/z)^k]/(1-z_0^k)$, we have

$$\varepsilon^P = \frac{k(k-1)z_0^k(z^P)^{1-k}}{kz_0^k(z^P)^{1-k} - 1 + (k-1)(1-z_0^k)a_H M}$$

and so $d\varepsilon^P/dz^P > 0$ at $M \to 0$. This in turn implies that $d\Lambda^P(\Omega)/d\Omega > 0$, since $d\xi/d\Omega > 0$ and $dz^P/d\Omega < 0$. Thus, as Ω rises from zero to one, Λ^P rises from $-a_H < \Lambda^P(0) < 0$ to $\Lambda^P(1) < 0$, where

$$\Lambda^P(1) = -\frac{a_H\bar{\varepsilon}}{1+\bar{\varepsilon}+\xi E_S/E_N}, \quad \text{where} \quad \bar{\varepsilon} \equiv \frac{\bar{z}^2 f(\bar{z})}{\int_{\bar{z}}^1 z dF(z) + a_H M},$$

because $\xi = 1$, $z^P = \bar{z}$ and $\varepsilon^P = \bar{\varepsilon}$ when $\Omega = 1$.

As we detail below, it matters how $\Lambda(0)$ compares to $\Lambda^P(0)$. Figure 6 shows two possible cases. In Figure 6a, $\Lambda(0) > \Lambda^P(0)$ and so, the sign of dz_{L0}/dM and dz_{L1}/dM depends on Ω. There exists $\bar{\Omega}$ such that $dz_{L0}/dM < 0$ and $dz_{L1}/dM > 0$ if $\Omega < \bar{\Omega}$; and $dz_{L0}/dM > 0$ and $dz_{L1}/dM < 0$ if $\bar{\Omega} < \Omega$. In Figure 6b, $\Lambda(0) < \Lambda^P(0)$ and so, $dz_{L0}/dM < 0$ and $dz_{L1}/dM > 0$ for any Ω.

From (A29), $\Lambda(0) = (w^{\sigma-1}-1)[2z_{H0}m^H(z_{H0}) - a_H m^H(a_H)] - a_H$ and $\Lambda(1) = -a_H$. Thus, $\Lambda^P(1) > \Lambda(1)$. As $\Lambda^P(0) < 0$, it follows that $\Lambda(0) > \Lambda^P(0)$ if $\Lambda(0) \geq 0$ or $(w^{\sigma-1}-1)[2z_{H0}m^H(z_{H0}) - a_H m^H(a_H)] \geq a_H$. This requires that $2z_{H0}m^H(z_{H0}) > a_H m^H(a_H)$ and so, implies that as Ω rises from zero to one, Λ falls at a constant rate. Hence, $\Lambda^P(\Omega)$ and $\Lambda(\Omega)$ intersect at a unique point in this case. Since $z_{H0}m^H(z_{H0}) > a_L m^H(a_L)$, the sufficient condition for $\Lambda(0) \geq 0$ is given by

$$(w^{\sigma-1}-1)[2a_L m^H(a_L) - a_H m^H(a_H)] \geq a_H.$$

When $\Lambda(0) < \Lambda^P(0)$, it is always the case that $\Lambda^P(\Omega) < \Lambda(\Omega)$, since $\Lambda(\Omega)$ is linear in Ω and $\Lambda^P(1) > \Lambda(1) = -1$. The condition $\Lambda(0) < \Lambda^P(0)$ requires the following:

$$(w^{\sigma-1}-1)[2z_{H0}m^H(z_{H0}) - a_H m^H(a_H)] - a_H < -\frac{a_H\varepsilon_0^P}{1+\varepsilon_0^P+\xi_0 E_S/E_N}, \tag{A32}$$

where $\xi_0 \equiv \xi(\Omega = 0)$ and $\varepsilon_0^P \equiv \varepsilon_0^P(\Omega = 0)$. It is true that $2z_{H0}m^H(z_{H0}) - a_H m^H(a_H) > 0$ if $a_L > a_H/2$. As $z_{H0}m^H(z_{H0}) < a_H m^H(a_H)$ and $\varepsilon_0^P < 1$, the condition $(w^{\sigma-1}-1)m^H(a_H) \leq 1/2$ is sufficient for (A32) to hold when $2z_{H0}m^H(z_{H0}) - a_H m^H(a_H) > 0$.

Proof of Proposition 5

From (A27), we find that $dz^P/dM = [F_M^L(G_{L0}^P G_{L1}^L - G_{L0}^L G_{L1}^P) - G_M^P(F_{L0}^L G_{L1}^L - F_{L1}^L G_{L0}^L)]/D$, where $D \equiv G_{L1}^L(F_{L0}^L G_P^P - F_P^L G_{L0}^P) + G_{L0}^L(F_P^L G_{L1}^P - F_{L1}^L G_P^P) > 0$ from the proof of Proposition 3. Thus, $dz^P/dM > 0$ if $F_M^L(G_{L0}^P G_{L1}^L - G_{L0}^L G_{L1}^P) > G_M^P(F_{L0}^L G_{L1}^L - F_{L1}^L G_{L0}^L)$, which using (A14)-(A15), (A18)-

(A19) and (A28) we simplify to obtain $\Lambda\xi^2 E_S/E_N < a_H(1+\Upsilon H)$, where

$$\Upsilon \equiv \frac{w^{\sigma-1}-1}{\Psi}\frac{z_{L0}m^L(z_{L0})}{d[z_{L0}m^L(z_{L0})]/dz_{L0}}, \qquad H \equiv -\frac{d[z_{L0}m^L(z_{L0})]/dz_{L0}}{d[z_{L1}m^L(z_{L1})]/dz_{L1}}\left(\frac{d\psi_S}{dz_{L1}}\right) - \frac{d\psi_S}{dz_{L0}} > 0. \quad \text{(A33)}$$

Because $z_{L0}\mu^L(z_{L0}) = z_{L1}\mu^L(z_{L1})$ and $(w^{\sigma-1}-1)\psi_S/\Psi = 1-\xi$, we have

$$\Upsilon H = (1-\xi)\left(\frac{\varepsilon_{L1}}{\epsilon_{L1}} + \frac{\varepsilon_{L0}}{\epsilon_{L0}}\right), \quad \text{where}$$

$$\varepsilon_{L1} \equiv \frac{d\psi_S}{dz_{L1}}\frac{z_{L1}}{\psi_S}, \qquad \varepsilon_{L0} \equiv -\frac{d\psi_S}{dz_{L0}}\frac{z_{L0}}{\psi_S}, \qquad \epsilon_{L1} \equiv -\frac{d[z_{L1}m^L(z_{L1})]}{m^L(z_{L1})dz_{L1}}, \qquad \epsilon_{L0} \equiv \frac{d[z_{L0}m^L(z_{L0})]}{m^L(z_{L0})dz_{L0}}.$$

Now, $\Lambda(\Omega)\xi^2 E_S/E_N < a_H(1+\Upsilon H)$ can be rewritten as follows:

$$\Lambda(\Omega) < \frac{a_H E_N}{\xi^2 E_S}\left[1 + (1-\xi)\left(\frac{\varepsilon_{L1}}{\epsilon_{L1}} + \frac{\varepsilon_{L0}}{\epsilon_{L0}}\right)\right].$$

Define

$$\Lambda^L(\Omega) \equiv \frac{a_H E_N}{\xi^2 E_S}\left[1 + (1-\xi)\left(\frac{\varepsilon_{L1}}{\epsilon_{L1}} + \frac{\varepsilon_{L0}}{\epsilon_{L0}}\right)\right]. \quad \text{(A34)}$$

Then $dz^P/dM > 0$ if $\Lambda(\Omega) < \Lambda^L(\Omega)$. Assuming $\pi_N(a_H) > \pi_S(a_H)$, which requires that $E_N/(\xi E_S) > (w^{\sigma-1}+1)\mu^H(a_H) - 1$, it is true that $\Lambda < \Lambda^L(\Omega)$. From (A34) we know that for any $\Omega < 1$, $\Lambda^L(\Omega) > a_H E_N/(\xi E_S) > a_H E_N/E_S = \Lambda^L(1)$. Thus $\Lambda(\Omega) \geq \Lambda^L(\Omega)$ if and only if $(w^{\sigma-1}-1)[2z_{H0}\mu^H(z_{H0}) - a_H\mu^H(a_H)] - a_H \geq a_H[(w^{\sigma-1}+1)\mu^H(a_H) - 1]$, which is not true since $z_{H0}\mu^H(z_{H0}) < a_H\mu^H(a_H)$. Hence, $\Lambda(\Omega) < \Lambda^L(\Omega)$ and so, $dz^P/dM > 0$.

Proof of Proposition 6

The Impact of IPRs

First, we show that $d\psi_N/d\Omega > 0$, $d\psi_S/d\Omega < 0$, and $d\Psi/d\Omega < 0$, where $\Psi \equiv \psi_N + (w^{\sigma-1}-1)\psi_S$. We find

$$\frac{d\psi_N}{d\Omega} = -z^P f(z^P)\frac{dz^P}{d\Omega},$$

$$\frac{d\psi_S}{d\Omega} = -\frac{\psi_S}{1-\Omega} + z_{L1}\mu^L(z_{L1})f(z_{L1})\frac{dz_{L1}}{d\Omega} - z_{L0}\mu^L(z_{L0})f(z_{L0})\frac{dz_{L0}}{d\Omega}.$$

From the proof of Proposition 3, we have $dz^P/d\Omega = [F^L_\Omega(G^L_{L1}G^P_{L0} - G^L_{L0}G^P_{L1}) - G^P_\Omega(F^L_{L0}G^L_{L1} - F^L_{L1}G^L_{L0})]/D$; $dz_{L0}/d\Omega = G^L_{L1}(F^L_P G^P_\Omega - F^L_\Omega G^P_P)/D$; and $dz_{L1}/d\Omega = -G^L_{L0}(F^L_P G^P_\Omega - F^L_\Omega G^P_P)/D$, where $D \equiv G^L_{L1}(F^L_{L0}G^P_P - F^L_P G^P_{L0}) + G^L_{L0}(F^L_P G^P_{L1} - F^L_{L1}G^P_P) > 0$. Using (A12)-(A21), substituting for $z^P f(z^P) = \varepsilon^P \psi_N / z^P$ from (A30) and simplifying, we obtain

$$\frac{dz^P}{d\Omega} = -\frac{z^P}{1-\Omega}\frac{\xi E_S}{E_N}\left[\frac{(w^{\sigma-1}-1)\psi_S}{\Psi} + \Upsilon H\right]\frac{1}{\tilde{D}}; \tag{A35}$$

$$\frac{d\psi_N}{d\Omega} = \frac{\varepsilon^P \psi_N}{1-\Omega}\frac{\xi E_S}{E_N}\left[\frac{(w^{\sigma-1}-1)\psi_S}{\Psi} + \Upsilon H\right]\frac{1}{\tilde{D}}; \tag{A36}$$

$$\frac{d\psi_S}{d\Omega} = -\frac{\psi_S}{1-\Omega}\left[1 + \frac{(1+\varepsilon^P)\psi_N}{(w^{\sigma-1}-1)\psi_S}\left(1 + \frac{\xi E_S}{E_N}\right)\frac{\Upsilon H}{\tilde{D}}\right] < 0; \tag{A37}$$

where Υ and H are given in (A33) and $\tilde{D} \equiv (1+\varepsilon^P)(1+\Upsilon H) + (1+\xi\varepsilon^P + \Upsilon H)\xi E_S/E_N$. Now using (A36)-(A37) and simplifying, we find that $d\Psi/d\Omega = d\psi_N/d\Omega + (w^{\sigma-1}-1)d\psi_S/d\Omega < 0$.

Second, we show that $dY_N/d\Omega > 0$ and $dY_S/\Omega < 0$. We have

$$Y_N = w\left(L_N - \int_{z^P}^{1} dF(z)\right) + \frac{E_N}{\sigma} + \left(\frac{\psi_N - \psi_S}{\Psi}\right)\frac{E_S}{\sigma}; \tag{A38}$$

$$Y_S = L_S(1-g_S^H) - \int_{z_{L0}}^{z_{L1}} dF(z) + \left(\frac{w^{\sigma-1}\psi_S}{\Psi}\right)\frac{E_S}{\sigma}. \tag{A39}$$

As $\sigma w = z^P(E_N + \xi E_S)/\psi_N$, we find that $dY_N/d\Omega > 0$ iff

$$w^{\sigma-1}\xi E_S \frac{\psi_S}{\Psi}\left(\frac{d\psi_N}{d\Omega}\frac{1}{\psi_N} - \frac{d\psi_S}{d\Omega}\frac{1}{\psi_S}\right) > (E_N + \xi E_S)\frac{\varepsilon^P}{z^P}\left(-\frac{dz^P}{d\Omega}\right).$$

Using (A35)-(A37), we simplify this inequality to $w^{\sigma-1} + \varepsilon^P > 0$, which is true. Next, because $\sigma = z_{L0}\mu^L(z_{L0})w^{\sigma-1}E_S/\Psi$, we find that $dY_S/d\Omega < 0$ iff

$$\frac{\psi_S\psi_N}{\Psi}\left(\frac{d\psi_N}{d\Omega}\frac{1}{\psi_N} - \frac{d\psi_S}{d\Omega}\frac{1}{\psi_S}\right) > z_{L0}\mu^L(z_{L0})f(z_{L0})\frac{dz_{L0}}{d\Omega} - z_{L1}\mu^L(z_{L1})f(z_{L1})\frac{dz_{L1}}{d\Omega}. \tag{A40}$$

Substituting for $dz_{L0}/d\Omega$ and $dz_{L1}/d\Omega$, we obtain

$$z_{L0}\mu^L(z_{L0})f(z_{L0})\frac{dz_{L0}}{d\Omega} - z_{L1}\mu^L(z_{L1})f(z_{L1})\frac{dz_{L1}}{d\Omega} = \frac{1}{1-\Omega}\frac{(1+\varepsilon^P)\psi_N}{w^{\sigma-1}-1}\left(1 + \frac{\xi E_S}{E_N}\right)\frac{\Upsilon H}{\tilde{D}}.$$

Now using (A36)-(A37) and simplifying, we find that the inequality (A40) holds.

Last, $dV_N/d\Omega > 0$ (since $dY_N/d\Omega > 0$ and $d\psi_N/d\Omega > 0$) and $dV_S/d\Omega < 0$ (since $dY_S/\Omega < 0$ and $d\Psi/d\Omega < 0$).

The Impact of Migration

First, we show that $d\psi_N/dM > 0$, $d\psi_S/dM < 0$, and $d\Psi/dM < 0$. We find that $d\psi_N/dM > 0$ iff $a_H > z^P f(z^P)(-dz^P/dM)$. From (A27), we find that $dz^P/dM = [F^L_M(G^P_{L0}G^L_{L1} - G^L_{L0}G^P_{L1}) - G^P_M(F^L_{L0}G^L_{L1} - F^L_{L1}G^L_{L0})]/D$, where $D \equiv G^L_{L1}(F^L_{L0}G^P_P - F^L_P G^P_{L0}) + G^L_{L0}(F^L_P G^P_{L1} - F^L_{L1}G^P_P) > 0$ from the proof of Proposition 3. Substituting for dz^P/dM, we simplify $a_H > z^P f(z^P)(-dz^P/dM)$ to obtain: $1 + (1 + 2\xi\varepsilon^P)\xi E_S/E_N + \Upsilon H\,(1 + \xi E_S/E_N) > 0$, which is true. Next, $d\psi_S/dM < 0$ if

$$\mu^H(a_H)a_H + \frac{d\psi_S}{dz_{H1}}\frac{dz_{H1}}{dM} + \frac{d\psi_S}{dz_{H0}}\frac{dz_{H0}}{dM} + \frac{d\psi_S}{dz_{L1}}\frac{dz_{L1}}{dM} + \frac{d\psi_S}{dz_{L0}}\frac{dz_{L0}}{dM} < 0.$$

From (A29), we have

$$\mu^H(a_H)a_H + \frac{d\psi_S}{dz_{H1}}\frac{dz_{H1}}{dM} + \frac{d\psi_S}{dz_{H0}}\frac{dz_{H0}}{dM} = -\frac{\Lambda(\Omega) + a_H}{w^{\sigma-1} - 1}.$$

From the proof of Proposition 4, we have $dz_{L0}/dM = G^L_{L1}(F^L_P G^P_M - F^L_M G^P_P)/D$ and $dz_{L1}/dM = -G^L_{L0}(F^L_P G^P_M - F^L_M G^P_P)/D$, where $D \equiv G^L_{L1}(F^L_{L0}G^P_P - F^L_P G^P_{L0}) + G^L_{L0}(F^L_P G^P_{L1} - F^L_{L1}G^P_P) > 0$. Using (A13)-(A21), (A28) and (A33), we obtain

$$\frac{d\psi_S}{dz_{L1}}\frac{dz_{L1}}{dM} + \frac{d\psi_S}{dz_{L0}}\frac{dz_{L0}}{dM} = \frac{1}{w^{\sigma-1} - 1}\left[\Lambda(\Omega)\left(1 + \varepsilon^P + \frac{\xi E_S}{E_N}\right) + a_H\varepsilon^P\right]\frac{\Upsilon H}{\tilde{D}}. \tag{A41}$$

Hence, $d\psi_S/dM < 0$ if $[\Lambda(\Omega) + a_H][1 + \varepsilon^P + (1 + \xi\varepsilon^P)\xi E_S/E_N] + a_H\Upsilon H(1 + \xi E_S/E_N) > 0$. This inequality holds when $\Lambda(\Omega) + a_H > 0$, which is true in Figure 6.

Next, $d\Psi/dM = d\psi_N/dM + (w^{\sigma-1} - 1)d\psi_S/dM$. We have

$$\frac{d\psi_N}{dM} = a_H\left[1 - \frac{\varepsilon^P}{\tilde{D}}\left(1 - \frac{\xi^2 E_S}{E_N} + \Upsilon H\right)\right], \tag{A42}$$

$$(w^{\sigma-1} - 1)\frac{d\psi_S}{dM} = -[\Lambda(\Omega) + a_H] + a_H\frac{\Upsilon H}{\tilde{D}}\left[\Lambda\left(1 + \varepsilon^P + \frac{\xi E_S}{E_N}\right) + \varepsilon^P\right]. \tag{A43}$$

It follows that $d\Psi/dM < 0$.

Second, using (A38), we find that $dY_N/dM > 0$, since $dz^P/dM > 0$, $d\psi_N/dM > 0$ and $d\psi_S/dM < 0$. Also using (A39), we find that $dY_S/dM < 0$ iff the following is true:

$$\frac{\psi_S\psi_N}{\Psi}\left(\frac{d\psi_N}{dM}\frac{1}{\psi_N} - \frac{d\psi_S}{dM}\frac{1}{\psi_S}\right) + \frac{d\psi_S}{dz_{L1}}\frac{dz_{L1}}{dM} + \frac{d\psi_S}{dz_{L0}}\frac{dz_{L0}}{dM} > 0. \tag{A44}$$

Note that if $dz_{L0}/dM > 0$ and $dz_{L1}/dM < 0$, then $dY_S/dM < 0$, since $d\psi_N/dM > 0$, $d\psi_S/dM < 0$,

$d\psi_S/dz_{L1} > 0$ and $d\psi_S/dz_{L0} < 0$. Using (A41)-(A43), we find that (A44) holds iff

$$1 - \frac{(w^{\sigma-1}-1)\psi_S}{\Psi}\frac{\varepsilon^P}{\tilde{D}}\left(1+\Upsilon H - \frac{\xi^2 E_S}{E_N}\right) + \xi\left\{\frac{\Lambda(\Omega)}{a_H}\left[1-\left(1+\varepsilon^P+\frac{\xi E_S}{E_N}\right)\frac{\Upsilon H}{\tilde{D}}\right] - \varepsilon^P\frac{\Upsilon H}{\tilde{D}}\right\} >$$

$$-\frac{\Lambda(\Omega)}{a_H}\left(1+\varepsilon^P+\frac{\xi E_S}{E_N}\right)\frac{\Upsilon H}{\tilde{D}} - \varepsilon^P\frac{\Upsilon H}{\tilde{D}},$$

which simplifies to the following:

$$[\Lambda(\Omega)+a_H]\left[(\xi+\Upsilon H)\left(1+\varepsilon^P+\frac{\xi E_S}{E_N}\right)+\varepsilon^P\frac{\xi^3 E_S}{E_N}\right] > -a_H(1-\xi)\left(1+\frac{\xi E_S}{E_N}+2\varepsilon^P\frac{\xi^2 E_S}{E_N}\right).$$

This inequality holds when $\Lambda(\Omega)+a_H > 0$, which is true in Figure 6.

Last, $dV_N/dM > 0$ (since $dY_N/dM > 0$ and $d\psi_N/dM > 0$) and $dV_S/dM < 0$ (since $dY_S/dM < 0$ and $d\Psi/dM < 0$).

Proof of Proposition 7

The Impact of IPRs

(i) $d\psi_N^q/d\Omega < 0$ since $(d\tilde{\psi}_N/d\Omega)/\tilde{\psi}_N > (d\psi_N/d\Omega)/\psi_N$, which simplifies to $\psi_N > z^P\tilde{\psi}_N$.

(ii) $d\psi_S^{Lq}/d\Omega > 0$ iff $(-d\tilde{\psi}_S^L/d\Omega)/\tilde{\psi}_S^L > (-d\psi_S^L/d\Omega)/\psi_S^L$, which requires the following:

$$\psi_S^L\left[f(z_{L0})\frac{dz_{L0}}{d\Omega} - \frac{m^L(z_{L1})}{m^L(z_{L0})}f(z_{L1})\frac{dz_{L1}}{d\Omega}\right] > z_{L0}\tilde{\psi}_S^L\left[f(z_{L0})\frac{dz_{L0}}{d\Omega} - f(z_{L1})\frac{dz_{L1}}{d\Omega}\right],$$

since $z_{L0}m^L(z_{L0}) = z_{L1}m^L(z_{L1})$. Substituting for $m^L(z_{L1})/m^L(z_{L0}) = z_{L0}/z_{L1}$, $f(z_{L1})/f(z_{L0}) = q^{k+1}$ where $q \equiv z_{L0}/z_{L1}$ and $e \equiv (-dz_{L1}/d\Omega)/(dz_{L0}/d\Omega) = (\alpha_{L1}-2\beta_{L1}z_{L0})/(2\beta_{L2}z_{L1}-\alpha_{L2}) > 0$, we obtain

$$\frac{\psi_S^L}{z_{L0}} - \tilde{\psi}_S^L\left(\frac{1+eq^{k+1}}{1+eq^{k+2}}\right) > 0, \tag{A45}$$

where

$$\frac{\psi_S^L}{z_{L0}} = \frac{(z_0/z_{L0})^k}{1-z_0^k}\left[\frac{k}{k-1}(\alpha_{L1}-\alpha_{L2}q^{k-1}) - \frac{k}{k-2}z_{L0}(\beta_{L1}-\beta_{L2}q^{k-2})\right],$$

$$\tilde{\psi}_S^L = \frac{(z_0/z_{L0})^k}{1-z_0^k}\left[\alpha_{L1}-\alpha_{L2}q^k - \frac{k}{k-1}z_{L0}(\beta_{L1}-\beta_{L2}q^{k-1})\right].$$

Note that (A45) holds when $e = 0$, in which case $z_{L0} = a_L$ and $z_{L1} \to a_L$, where $a_L = \alpha_{L1}/(2\beta_{L1}) > \alpha_{L2}/(2\beta_{L2})$ and $a_L \to (\alpha_{L2}-\alpha_{L1})/(\beta_{L2}-\beta_{L1})$. Further, the left hand side in (A45) falls as e rises from $e = 0$, in which case z_{L0} falls from $z_{L0} = a_L$, z_{L1} rises from

$z_{L1} \to a_L$, and q falls from $g \to 1$. This is because $(1+eq^{k+1})/(1+eq^{k+2})$ rises as e rises and q falls from $q \to 1$, while ψ_S^L/z_{L0} falls as a faster rate than $\tilde{\psi}_S^L$ does as q falls from $q \to 1$ and z_{L0} falls from $z_{L0} = a_L$:

$$-\frac{d(\psi_S^L/z_{L0})}{dq} > -\frac{d\tilde{\psi}_S^L}{dq} \quad \text{and} \quad -\left.\frac{d(\psi_S^L/z_{L0})}{dz_{L0}}\right|_{q\to 1} > -\left.\frac{d\tilde{\psi}_S^L}{dz_{L0}}\right|_{q\to 1}.$$

Last, note that $e < 1$. It must be true that $de/dz_{L0} < 0$. This requires that $\beta_{L2}(\alpha_{L1} - 2\beta_{L1}z_{L0})(-dz_{L1}/dz_{L0}) - \beta_{L1}(2\beta_{L2}z_{L1} - \alpha_{L2}) < 0$ or $e^2 < \beta_{L1}/\beta_{L2} < 1$, since $dz_{L1}/dz_{L0} = -e$, $z_{L0}(\alpha_{L1} - \beta_{L1}z_{L0}) - z_{L1}(\alpha_{L2} - \beta_{L2}z_{L1}) = 0$ and $\beta_{L1}/\beta_{L2} < 1$.

It follows that (A45) holds for any $e < 1$ if it holds at $e \to 1$, which requires the following:

$$\frac{\psi_S^L}{z_{L0}} - \tilde{\psi}_S^L\left(\frac{1+q^{k+1}}{1+q^{k+2}}\right) > 0.$$

The left hand side of this inequality falls as q falls for any z_{L0} and so, we can set $q = 0$ to obtain the sufficient condition: $\alpha_{L1} - \beta_{L1}z_{L0}k/(k-2) > 0$, which holds for $k \geq 4$, since $z_{L0} < a_L \leq \alpha_{L1}/(2\beta_{L1})$ from Lemma 1.

(iii) $d\psi_S^{Hq}/d\Omega = 0$ since $(-d\psi_S^H/d\Omega)/\psi_S^H = (-d\tilde{\psi}_S^H/d\Omega)/\tilde{\psi}_S^H = 1/(1-\Omega)$.

The Impact of Migration

(i) $d\psi_N^q/dM > 0$ iff $(d\psi_N/dM)/\psi_N > (d\psi_N^o/dM)/\tilde{\psi}_N$, which requires the following:

$$(\psi_N - z^P\tilde{\psi}_N)f(z^P)\frac{dz^P}{dM} + \int_{z^P}^1 (a_H - z)dF(z) > 0.$$

The first term is positive. The second is positive iff $a_H[(z^P)^{-k} - 1] - [(z^P)^{1-k} - 1]k/(k-1) > 0$, which is true if $k > 3$ and $a_H > 1/2$. This is because $\int_{z^P}^1 (a_H - z)dF(z) > 0$ for $k = 3$ when $a_H > 1/2$ (because $z^P < a_H$) and it further rises as k rises. Lemma 1 implies that $a_H > 1/2$ if $\alpha_{H2}/\beta_{H2} > 1$ or $m^H(1) > 0$.

(ii) First, we show that $d\psi_S^{Lq}dM > 0$ if the L imitation set contracts. We note that $d\psi_S^{Lq}dM > 0$ iff $(-d\tilde{\psi}_S^L/dM)/\tilde{\psi}_S^L > (-d\psi_S^L/dM)/\psi_S^L$, which requires the following:

$$\psi_S^L\left[f(z_{L0})\frac{dz_{L0}}{dM} - \frac{m^L(z_{L1})}{m^L(z_{L0})}f(z_{L1})\frac{dz_{L1}}{dM}\right] > z_{L0}\tilde{\psi}_S^L\left[f(z_{L0})\frac{dz_{L0}}{dM} - f(z_{L1})\frac{dz_{L1}}{dM}\right],$$

where $dz_{L0}/dM > 0$ and $dz_{L1}/dM < 0$. Since $e \equiv (-dz_{L1}/d\Omega)/(dz_{L0}/d\Omega) = (-dz_{L1}/dM)/(dz_{L0}/dM)$, we obtain (A45), which holds for $k \geq 4$.

Second, $d\psi_S^{Lq} dM < 0$ if the L imitation set expands. Note that $d\psi_S^{Lq} dM < 0$ iff $(-d\tilde{\psi}_S^L/dM)/\tilde{\psi}_S^L < (-d\psi_S^L/dM)/\psi_S^L$, which requires the following:

$$\psi_S^L \left[-f(z_{L0})\frac{dz_{L0}}{dM} + \frac{m^L(z_{L1})}{m^L(z_{L0})} f(z_{L1})\frac{dz_{L1}}{dM} \right] > z_{L0}\tilde{\psi}_S^L \left[-f(z_{L0})\frac{dz_{L0}}{dM} + f(z_{L1})\frac{dz_{L1}}{dM} \right],$$

where $dz_{L0}/dM < 0$ and $dz_{L1}/dM < 0$. Again, we obtain (A45), which holds for $k \geq 4$.

(iii) $d\psi_S^{Hq}/dM > 0$ if $(d\psi_S^H/dM)/\psi_S^H - (d\tilde{\psi}_S^H/dM)/\tilde{\psi}_S^H > 0$, which requires the following:

$$\psi_S^H \left[f(z_{H0})\frac{dz_{H0}}{dM} - \frac{m^H(z_{H1})}{m^H(z_{H0})} f(z_{H1})\frac{dz_{H1}}{dM} \right] >$$

$$z_{H0}\tilde{\psi}_S^H \left[f(z_{H0})\frac{dz_{H0}}{dM} - f(z_{H1})\frac{dz_{H1}}{dM} \right] + (\psi_S^H - a_H\tilde{\psi}_S^H)\frac{m^H(a_H)}{m^H(z_{H0})}.$$

We note that

$$\psi_S^H - a_H\tilde{\psi}_S^H = -\frac{(z_0/z_{H0})^k}{1-z_0^k} \left[a_H(1-q^k) - \frac{k}{k-1} z_{H0}(1-q^{k-1}) \right] < 0,$$

where the term in square brackets is positive when $k = 2$ (since $a_H > z_{H0}$) and rises as k rises. Thus, it remains to show the following:

$$\psi_S^H \left[f(z_{H0})\frac{dz_{H0}}{dM} - \frac{m^H(z_{H1})}{m^H(z_{H0})} f(z_{H1})\frac{dz_{H1}}{dM} \right] > z_{H0}\tilde{\psi}_S^H \left[f(z_{H0})\frac{dz_{H0}}{dM} - f(z_{H1})\frac{dz_{H1}}{dM} \right].$$

Substituting for $m^H(z_{H1})/m^H(z_{H0}) = z_{H0}/z_{H1}$, $f(z_{H1})/f(z_{H0}) = q^{k+1}$, where $q \equiv z_{H0}/z_{H1}$ and $e \equiv (-dz_{H1}/dM)/(dz_{H0}/dM) = (\alpha_{H1} - 2\beta_{H1}z_{L0})/(2\beta_{H2}z_{H1} - \alpha_{H2}) > 0$, we obtain (A45) where $j = L$ is replaced with $j = H$. We have shown that (A45) holds for $k \geq 4$ when $j = L$; likewise, it holds for $k \geq 4$ when $j = H$.